Chapter 10

Areas and Volumes

Learning Objectives

After completing this chapter, you will be able to

- find the areas of simple polygons.
- understand and apply the relations between sides and areas of similar figures.
- find circumferences of circles.
- explore and apply the formula for the area of a circle.
- find the arc lengths and the areas of sectors.
- understand and apply the formulae for the volume and surface area of a prism and cylinder.
- realize the accumulation of errors caused by the application of formulae.
- appreciate the past attempts to find the approximate values of π.

The 4 different regions of an archery target are of the same area and have the same centre shown in the figure. If the radius of the red region is 10 cm, find the width of each region, correct to 1 decimal place.

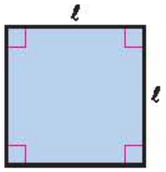
Preview

[Basic knowledge and techniques required for this chapter.]

A. Basic Knowledge

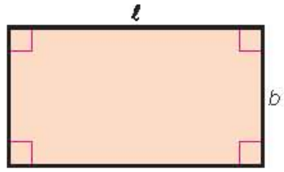
1. Areas of plane figures

(a)



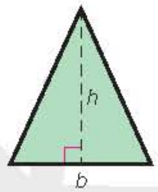
Area of square $= \ell^2$

(b)



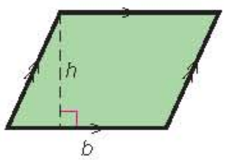
Area of rectangle $= \ell b$

(c)



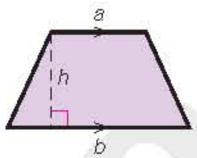
Area of triangle $= \frac{1}{2}bh$

(d)



Area of parallelogram $= bh$

(e)



Area of trapezium $= \frac{1}{2}(a+b)h$

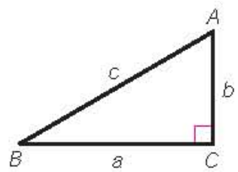
2. Pythagoras' theorem

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

i.e. In ΔABC, if $\angle C = 90°$,

then $a^2 + b^2 = c^2$.

[**Abbreviation:** Pyth. theorem]



3. For an approximate value x (correct to the nearest N),

maximum absolute error of $x = \frac{N}{2}$

upper limit of the actual value $= x + \frac{N}{2}$

lower limit of the actual value $= x - \frac{N}{2}$

B. Basic Technique

1. Substitution

Example: It is given that $S = \frac{n(a+\ell)}{2}$. If $n = 5$, $a = 3$ and $\ell = 7$, find the value of S.

Solution: $S = \frac{5(3+7)}{2}$

$= \underline{\underline{25}}$

2. Simplification of ratios

Example: Simplify $39:9$.

Solution: $$39:9=\frac{39}{9}$$
$$=\frac{13}{3}$$
$$=\underline{\underline{13:3}}$$

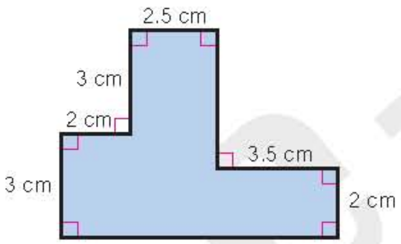
10.1 Areas of Polygons

In primary school, we have learned how to find the areas of squares, rectangles, triangles, parallelograms and trapeziums. Now we are going to find the areas of some simple polygons by using relevant formulae together.

Additional Example 10.1

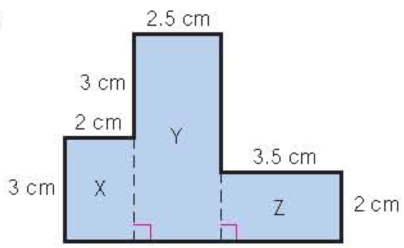
Example 10.1 Finding the area of a polygon using the formula for the area of a rectangle

Find the area of the following polygon.



Solution

Divide the polygon into three rectangles X, Y and Z.



Area of X $= 2\times 3\ \text{cm}^2$
$= 6\ \text{cm}^2$

Area of Y $= 2.5\times(3+3)\ \text{cm}^2$
$= 15\ \text{cm}^2$

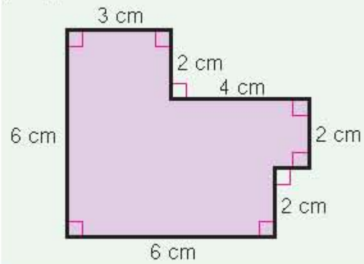
Area of Z $= 3.5\times 2\ \text{cm}^2$
$= 7\ \text{cm}^2$

$\therefore$ Area of the polygon $= (6+15+7)\ \text{cm}^2$
$= \underline{\underline{28\ \text{cm}^2}}$

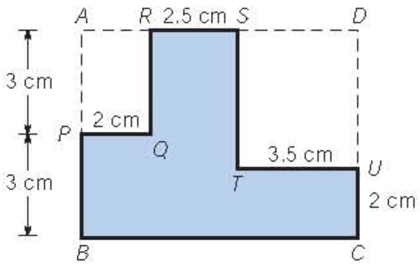
Classwork 10.1

Solution

Find the area of the following polygon.



Alternative method:



Area of $APQR = 2\times 3\text{ cm}^2$

$= 6\text{ cm}^2$

Area of $STUD = 3.5\times(3+3-2)\text{ cm}^2$

$= 14\text{ cm}^2$

Area of $ABCD = (2+2.5+3.5)\times(3+3)\text{ cm}^2$

$= 48\text{ cm}^2$

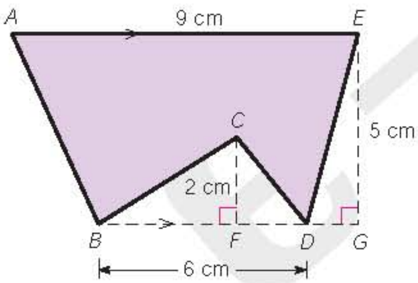
$\therefore$ Area of the polygon $= (48-6-14)\text{ cm}^2$

$= \underline{\underline{28\text{ cm}^2}}$

Additional Example 10.2

Example 10.2 Finding the area of a polygon using different formulae for areas

Find the area of the shaded region.



Solution

Area of trapezium $ABDE = \dfrac{1}{2}\times(9+6)\times 5\text{ cm}^2$

$= 37.5\text{ cm}^2$

Area of $\Delta BCD = \dfrac{1}{2}\times 6\times 2\text{ cm}^2$

$= 6\text{ cm}^2$

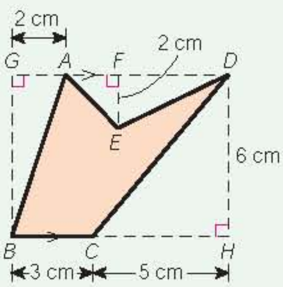
$\therefore$ Area of the shaded region $= (37.5-6)\text{ cm}^2$

$= \underline{\underline{31.5\text{ cm}^2}}$

Classwork 10.2

Solution

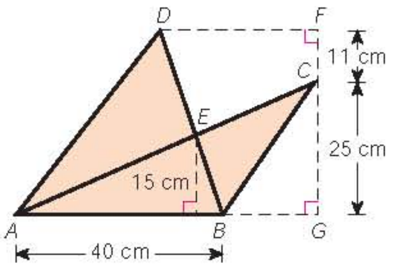
Find the area of the shaded region.



Additional Example 10.3

Example 10.3 Finding the area of a polygon using the formula for the area of a triangle

In the figure, AC and DB meet at E. Find the area of the shaded region.



Solution

Area of $\Delta ABC = \frac{1}{2}\times 40\times 25 \text{ cm}^2$

$= 500 \text{ cm}^2$

Area of $\Delta ABD = \frac{1}{2}\times 40\times 36 \text{ cm}^2$

$= 720 \text{ cm}^2$

Taking AB as the base, the height of ΔABD is $(11+25)$ cm, i.e. 36 cm.

Area of $\Delta ABE = \frac{1}{2}\times 40\times 15 \text{ cm}^2$

$= 300 \text{ cm}^2$

$\therefore$ Area of the shaded region $= (500+720-300) \text{ cm}^2$

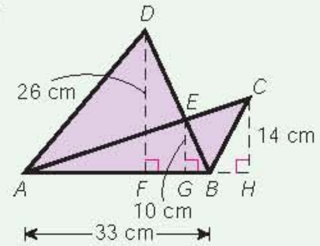
$= \underline{\underline{920 \text{ cm}^2}}$

Classwork 10.3

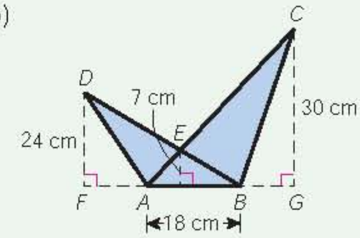
Solution

In each of the following figures, AC and DB meet at E. Find the areas of the shaded regions.

(a)



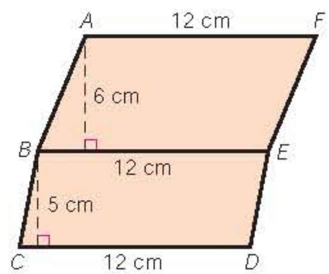
(b)



Skills Upgrading Corner 10.1

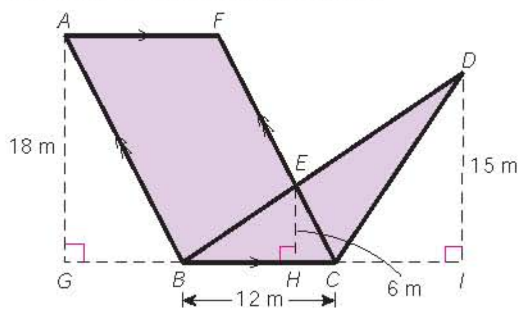
Solution

1. In the figure, $ABEF$ and $BCDE$ are parallelograms. Find the area of the figure.



Solution

2. In the figure, $ABCF$ is a parallelogram. FC and BD meet at E. Find the area of the shaded region.

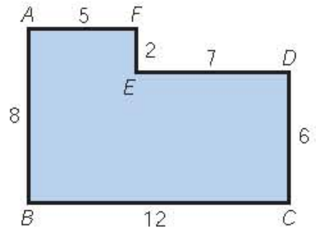


Exercise 10A

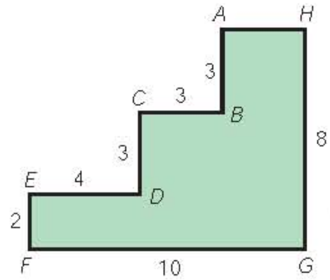
Level 1

1. Each of the following figures is formed by rectangles. Find the area of each figure.

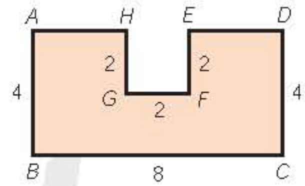
(a)



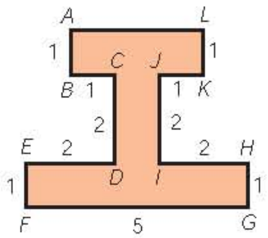
(b)



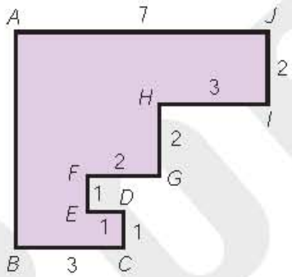
(c)



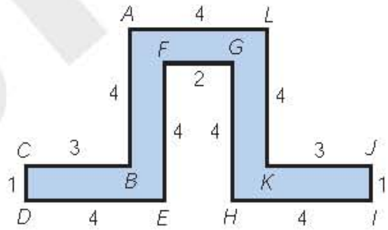
(d)



(e)

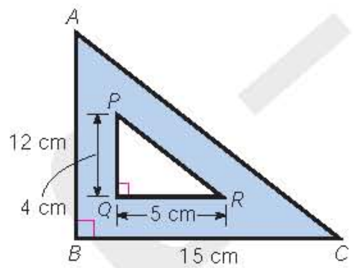


(f)

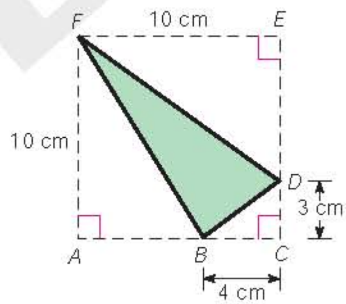


2. Find the area of the shaded region in each of the following figures.

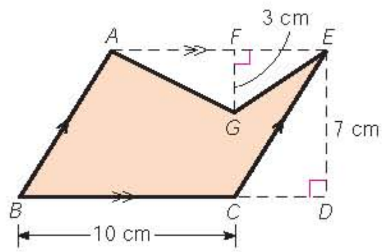
(a)



(b)

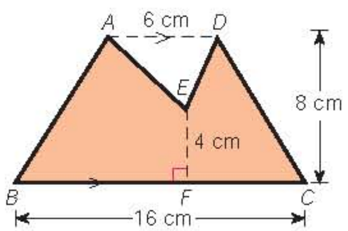


(c)

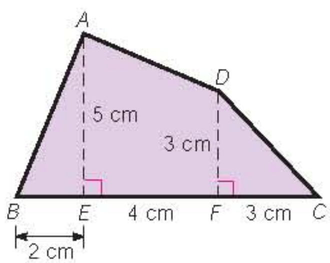


3. Find the area of the shaded region in each of the following figures.

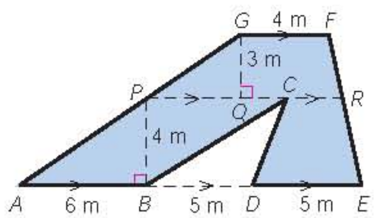
(a)



(b)



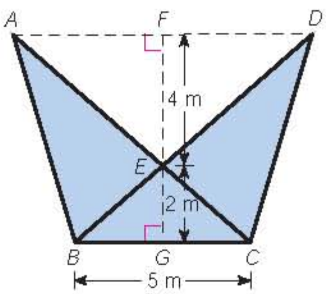
(c)



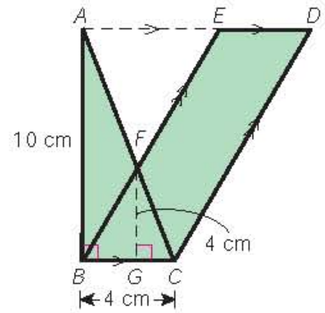
Level 2

4. Find the area of the shaded region in each of the following figures.

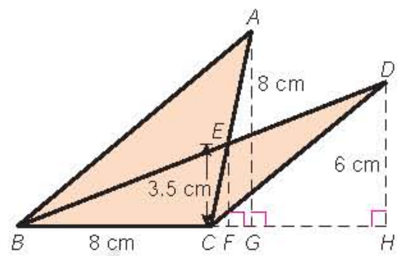
(a) AC and BD meet at E.



(b) AC and BE meet at F.



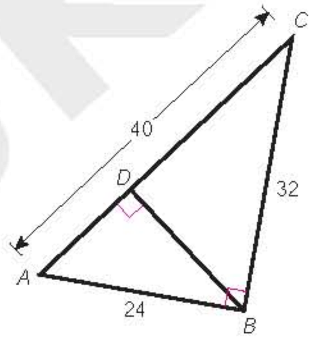
(c) AC and BD meet at E.



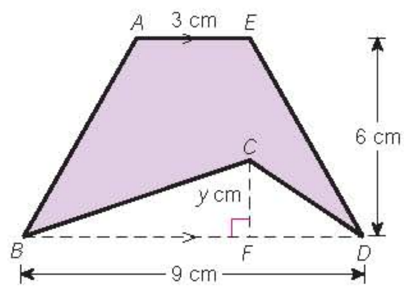
5. The figure shows ΔABC where $\angle ABC = 90°$. D is a point on AC such that $BD \perp AC$. $AB = 24$, $BC = 32$ and $AC = 40$.

(a) Find the area of ΔABC.

(b) Find the length of BD.



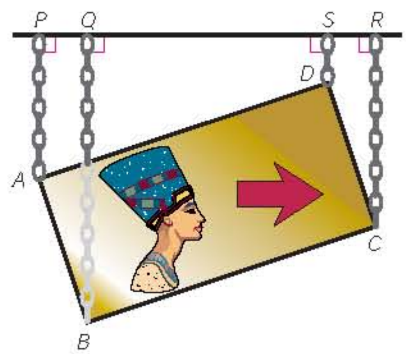
6. In the figure, if the area of the shaded region is 27 cm^2, find the value of y.



7. The figure shows a rectangular board $ABCD$ held by 4 vertical chains AP, BQ, CR and DS below the ceiling. Given that $AP = 30$ cm, $BQ = 60$ cm, $CR = 40$ cm, $DS = 10$ cm, $PQ = RS = 10$ cm and $QS = 50$ cm,

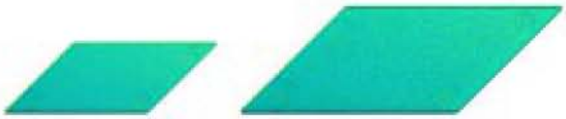
(a) find the areas of trapeziums $ABQP$ and $BCRQ$.

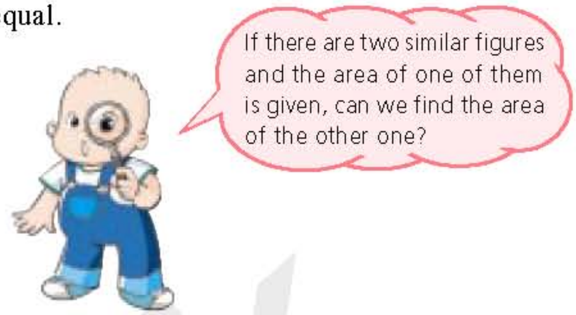
(b) Hence, find the area of $ABCD$.



10.2 Areas of Similar Figures

Two figures of the same shape are similar figures. Their corresponding sides are in proportion and their corresponding angles are equal.



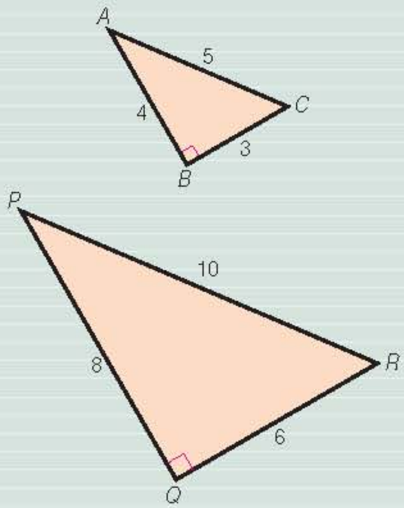


Class Activity 10.1

Aim: To explore the ratio of the areas of similar figures

1. Consider the pair of similar triangles as shown.

Show Ans.



(a) $AB : PQ =$ ______ : ______

$=$ ______ : ______

(b) Area of $\Delta ABC =$ ____________

Area of $\Delta PQR =$ ____________

(c) Area of ΔABC : Area of $\Delta PQR =$ ______ : ______

$=$ ______ : ______

2. Consider the following pair of similar figures.

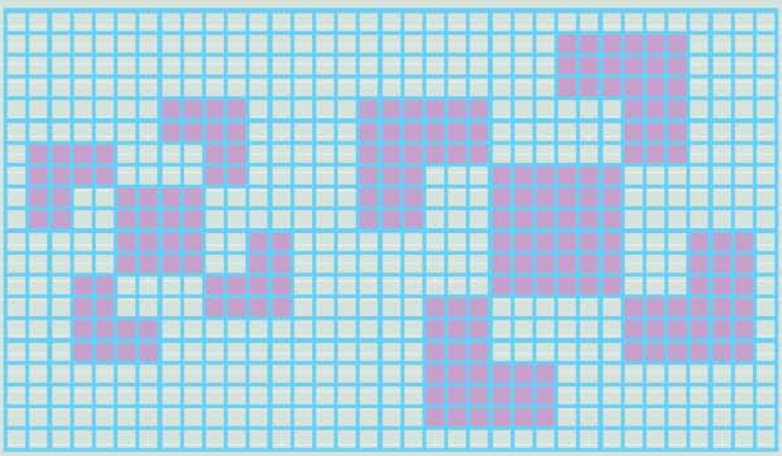


Figure I

Figure II

Show Ans.

(a) Ratio of the lengths of the corresponding sides of Figure I and Figure II = ________________

(b) Area of Figure I = ________________

Area of Figure II = ________________

(c) Ratio of the areas of Figure I and Figure II = ________________

3. For two similar figures, what is the relation between the ratio of the lengths of their corresponding sides and the ratio of their areas?

Show Ans.

____

____

Now I see ...

The ratio of the areas of two similar figures is the square of the ratio of the lengths of their corresponding sides.

In fact,

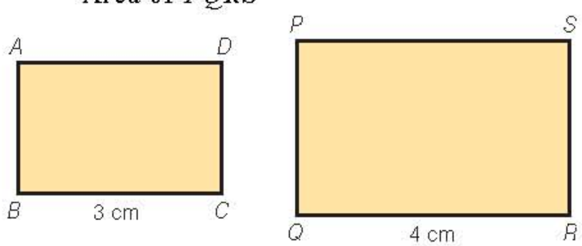
> For two similar figures, if the lengths of their corresponding sides are ℓ_1 and ℓ_2 and their areas are A_1 and A_2 respectively, then $\dfrac{A_1}{A_2}=(\dfrac{\ell_1}{\ell_2})^2$.

Extension 10.1

Solution

1. The following are two similar figures, where the lengths of corresponding sides BC and QR are 3 cm and 4 cm respectively.

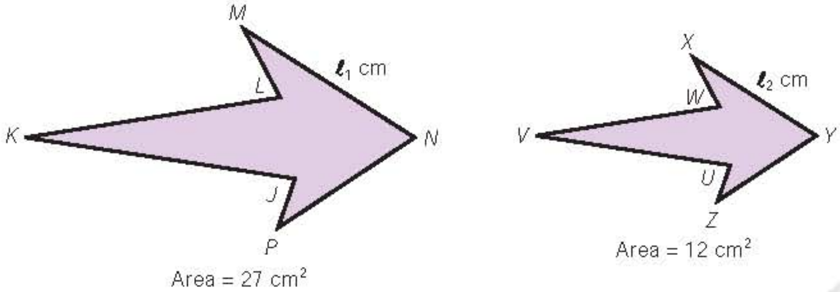
Find $\dfrac{\text{Area of } ABCD}{\text{Area of } PQRS}$.



Solution

2. The following are two similar figures, where the lengths of corresponding sides MN and XY are ℓ_1 cm and ℓ_2 cm respectively.

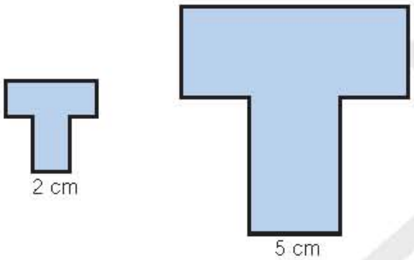
Find $\frac{\ell_1}{\ell_2}$.



Additional Example 10.4

Example 10.4 Finding the area of similar figures

The figure shows two similar letters T. If the area of the smaller T is 16 cm^2, find the area of the larger T.



Solution

$$\frac{\text{Area of smaller T}}{\text{Area of larger T}} = \left(\frac{2}{5}\right)^2$$

$$\frac{16 \text{ cm}^2}{\text{Area of larger T}} = \frac{2}{5} \times \frac{2}{5}$$

$$\frac{16 \text{ cm}^2}{\text{Area of larger T}} = \frac{4}{25}$$

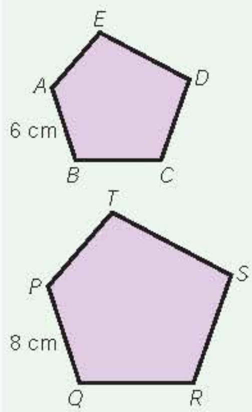
$$\therefore \quad \text{Area of larger T} = \frac{25}{4} \times 16 \text{ cm}^2$$

$$= \underline{\underline{100 \text{ cm}^2}}$$

Classwork 10.4

Solution

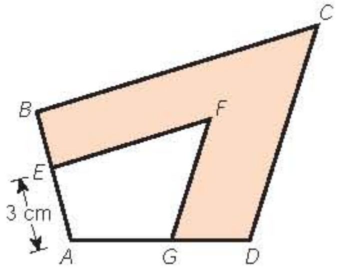
In the figure, $ABCDE$ and $PQRST$ are two similar figures. If the area of $PQRST$ is 96 cm^2, find the area of $ABCDE$.



Additional Example 10.5

Example 10.5 Finding the ratio of areas of similar figures

In the figure, $ABCD$ and $AEFG$ are similar figures. The areas of $AEFG$ and the shaded region are 18 cm^2 and 32 cm^2 respectively.



(a) Find the ratio of the areas of $AEFG$ and $ABCD$.

(b) Find the length of BE.

Solution

(a) Ratio of the areas of $AEFG$ and $ABCD$

$$= 18 \text{ cm}^2 : (18+32) \text{ cm}^2$$
$$= 18 : 50$$
$$= \underline{\underline{9 : 25}}$$

(b)
$$\frac{\text{Area of } AEFG}{\text{Area of } ABCD} = \left(\frac{AE}{AB}\right)^2$$
$$\frac{9}{25} = \left(\frac{3 \text{ cm}}{AB}\right)^2$$
$$\frac{9}{25} = \frac{3 \text{ cm}}{AB} \times \frac{3 \text{ cm}}{AB}$$
$$\frac{9}{25} = \frac{9 \text{ cm}^2}{AB^2}$$
$$AB^2 = \frac{25}{9} \times 9 \text{ cm}^2$$
$$AB^2 = 25 \text{ cm}^2$$
$$AB = 5 \text{ cm}$$

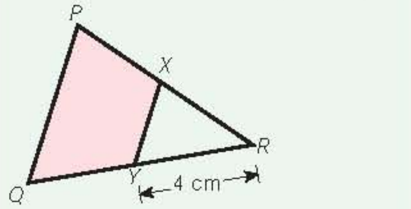
◂ 5 is the positive square root of 25.

$$\therefore \quad BE = (5-3) \text{ cm}$$
$$= \underline{\underline{2 \text{ cm}}}$$

Classwork 10.5

Solution

In the figure, $\Delta PQR \sim \Delta XYR$. The areas of ΔXYR and the shaded region are 24 cm^2 and 49.5 cm^2 respectively.



(a) Find the ratio of the areas of ΔXYR and ΔPQR.

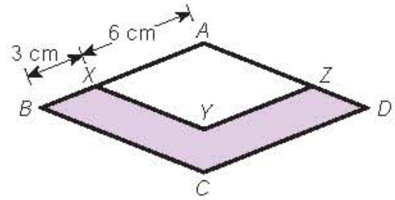
(b) Find the length of QY.

Skills Upgrading Corner 10.2

Solution

1. The figure shows two similar quadrilaterals $ABCD$ and $AXYZ$. If the area of $AXYZ$ is 36 cm^2, find the area of the shaded region.

◂



Solution

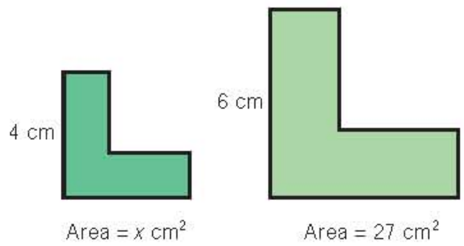
2. It is known that the scale of a map is 1 cm to 2 km. What is the actual area of a country park if its area on the map is 3 cm^2?

Exercise 10B

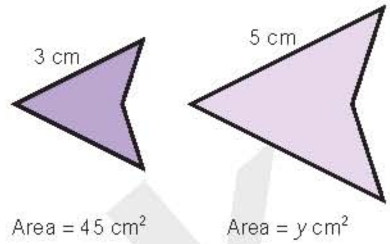
Level 1

Find the unknown in each of the following pairs of similar figures. **(1 – 2)**

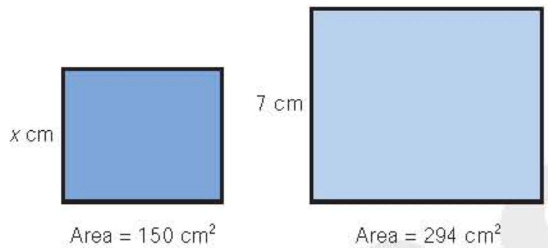
1. **(a)**



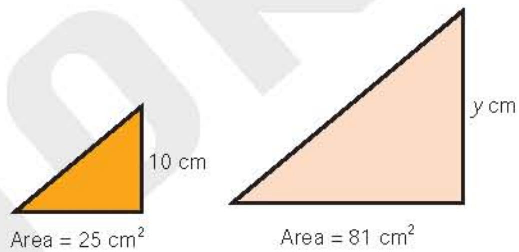
(b)



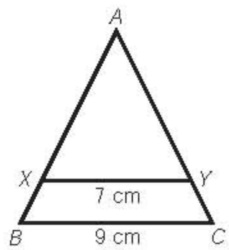
2. **(a)**



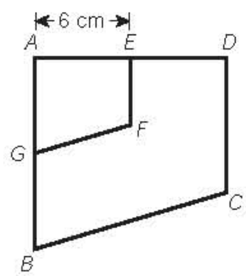
(b)



3. In the figure, $\Delta ABC \sim \Delta AXY$ and the area of ΔABC is 162 cm^2. Find the area of ΔAXY.

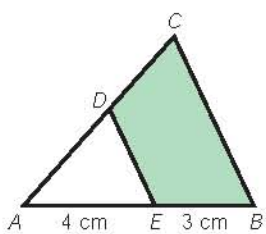


4. The figure shows two similar quadrilaterals $ABCD$ and $AGFE$. Their areas are 48 cm^2 and 27 cm^2 respectively. If $AE = 6\text{ cm}$, find the length of AD.

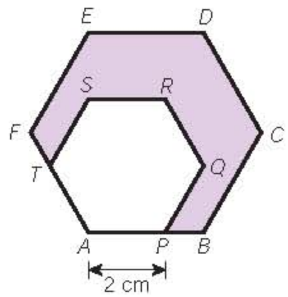


Level 2

5. In the figure, $\Delta ABC \sim \Delta AED$ and the area of ΔABC is 147 cm^2. Find the area of the shaded region.



6. In the figure, $ABCDEF$ and $APQRST$ are regular hexagons. If the areas of $APQRST$ and the shaded region are 60 cm^2 and 75 cm^2 respectively,
   (a) find the ratio of the areas of $APQRST$ and $ABCDEF$.
   (b) find the length of PB.



7. Two pieces of copper wire with different lengths are bent into two regular pentagons, where the length of the shorter wire is $\frac{4}{7}$ of the longer one. If the area of the larger regular pentagon is 49 cm^2, find the area of the smaller regular pentagon.

8. The scale of the floor plan of a flat is 1 cm to 2 m. If the area of the master bedroom on the floor plan is 7.5 cm^2, find its actual area.

9. The scale of the floor plan of a construction site is 1 cm to 30 m. If the actual area of a building is $3\,330\text{ m}^2$, find its area on the floor plan.

10.3 Circumferences of Circles

Figure 10.1(a) shows a **circle**. The boundary of the circle is called the **circumference**. The line segment joining the **centre** to any point on the circumference is the **radius** of the circle (see Figure 10.1(b)). The line segment joining any two points on the circumference and passing through the centre is the **diameter** of the circle (see Figure 10.1 (c)).

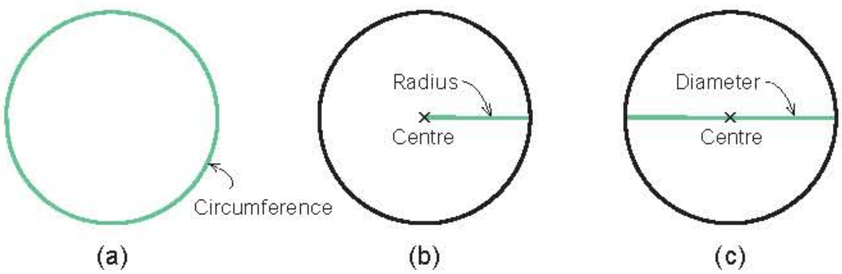


Figure 10.1

circle 圓　circumference 圓周　centre 圓心　radius 半徑　diameter 直徑

The ratio of the circumference to the diameter is the same for all circles. The ratio is called pi, denoted by the Greek letter π.

The Greek letter π was introduced by William Jones, a British mathematician, to represent the ratio. This symbol is still in use nowadays.

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

or $\quad$ Circumference $= \pi \times$ Diameter

In symbols,

$$C = \pi d \quad \text{or} \quad C = 2\pi r$$

where C is the circumference, d is the diameter and r is the radius of the circle.

We usually obtain an approximate value of π by pressing the key π of a calculator, or we may take it as $\frac{22}{7}$, 3.14, ⋯ etc. according to the requirements of questions.

Additional Example 10.6

Example 10.6 Finding the circumference with given diameter

Find the circumference of a circle with the diameter of 6.4 cm. Give your answer correct to 1 decimal place.

Solution

$$\begin{aligned} \text{Circumference} &= \pi \times 6.4 \text{ cm} \\ &= \underline{\underline{20.1 \text{ cm}}} \text{ (corr. to 1 d.p.)} \end{aligned}$$

Classwork 10.6 Solution

Find the circumferences of the following circles using the information given. (Give your answers correct to 1 decimal place.)

(a) Diameter = 15.2 cm

(b) Radius = 8.71 cm

Additional Example 10.7

Example 10.7 Finding the radius with given circumference

The figure shows a piece of circular cookie. Its circumference is 15.4 cm. Find its radius. (Take $\pi = \frac{22}{7}$.)

Solution

Let r cm be the radius of the cookie.

$$2 \times \frac{22}{7} \times r = 15.4$$

$$r = \frac{15.4 \times 7}{44}$$

$$= 2.45$$

$\therefore$ $\underline{\underline{\text{The radius of the cookie is 2.45 cm.}}}$

Classwork 10.7 Solution

If the circumference of a circle is 35.2 cm, find its

(a) diameter.

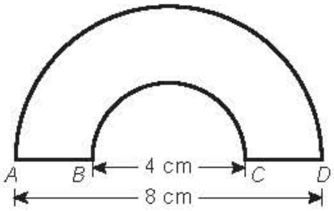
(b) radius.

(Take $\pi = \frac{22}{7}$.)

Additional Example 10.8

Example 10.8 Finding the perimeter of a figure formed by circles and straight lines

The figure is formed by semi-circles and straight lines. Find its perimeter, correct to the nearest 0.1 cm.



Solution

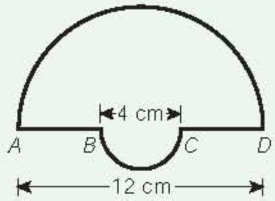
$AB + CD = AD - BC$
$= (8 - 4)$ cm
$= 4$ cm

Perimeter of the figure
$= \frac{1}{2}$(Circumference of the larger circle)
$+ \frac{1}{2}$(Circumference of the smaller circle) $+ AB + CD$
$= (\frac{1}{2} \times \pi \times 8 + \frac{1}{2} \times \pi \times 4 + 4)$ cm
$= \underline{\underline{22.8 \text{ cm}}}$ (corr. to the nearest 0.1 cm)

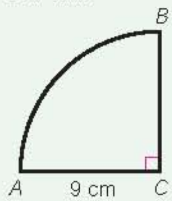
Classwork 10.8

Solution

(a) The figure is formed by semi-circles and straight lines. Find its perimeter, correct to the nearest 0.1 cm.



(b) The figure is formed by $\frac{1}{4}$ of a circle and straight lines. Find its perimeter, correct to the nearest 0.1 cm.



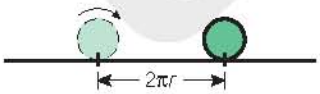
Additional Example 10.9

Example 10.9 Problems involving circumference

The radius of a car wheel is 30 cm. How far does it travel in 5 000 revolutions of the wheel? (Take $\pi = 3.14$.)

Solution

[Analysis:



The distance travelled in one revolution of the wheel = $2\pi r$]

Distance travelled in 1 revolution of the wheel
$=$ Circumference of the wheel
$= 2 \times 3.14 \times 30$ cm
$= 188.4$ cm

$\therefore$ Distance travelled in 5 000 revolutions of the wheel
$= 5\,000 \times 188.4$ cm
$= 942\,000$ cm
$= \underline{\underline{9.42 \text{ km}}}$

Classwork 10.9

Solution

(a) The diameter of a coin is 2.5 cm. How far does it move in 40 revolutions? (Take $\pi = 3.14$.)

(b) When the wheel of a bicycle makes 30 revolutions, the bicycle travels 66 m. Find the diameter of the wheel. (Take $\pi = \frac{22}{7}$.)

Additional Example 10.10

Example 10.10 Comparison of circumferences

The difference between the diameters of two circles is 5 cm. Find the difference between their circumferences. (Express your answer in terms of π.)

Solution

Let d_1 cm and d_2 cm be the diameters of the larger and smaller circles respectively.

$\therefore \quad d_1 - d_2 = 5$

Circumference of the larger circle $= \pi d_1$ cm

Circumference of the smaller circle $= \pi d_2$ cm

$$\begin{aligned}\therefore \quad \text{Difference between their circumferences} &= (\pi d_1 - \pi d_2) \text{ cm} \\ &= \pi(d_1 - d_2) \text{ cm} \\ &= \underline{\underline{5\pi \text{ cm}}}\end{aligned}$$

Classwork 10.10

Solution

(a) The difference between the radii of two circles is 8 cm. Find the difference between their circumferences. (Express your answer in terms of π.)

(b) Given that the difference between the circumferences of two circles is 4 m, find the difference between their diameters. (Express your answer in terms of π.)

Skills Upgrading Corner 10.3

Solution

1. The length of the minute-hand of a clock is 15 cm. How far does the tip of the minute-hand move in 45 minutes? (Express your answer in terms of π.)

Solution

2. The ratio of the diameters of two circles is 5 : 2. What is the ratio of their circumferences?

Exercise 10C

[In this exercise, give your answers correct to 1 decimal place if necessary.]

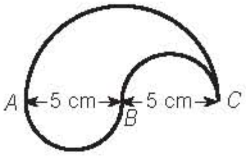
Level 1

1. Complete the following table. ($\pi = \frac{22}{7}$.)

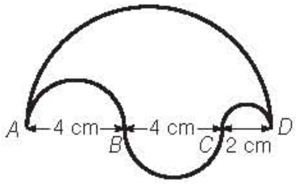
	Radius	Diameter	Circumference
(a)	7 cm		
(b)	21 cm		
(c)		21 cm	
(d)		35 cm	
(e)			17.6 cm
(f)			103.4 cm

2. The following figures are formed by semi-circles, find the perimeter of each figure. (Express your answers in terms of π.)

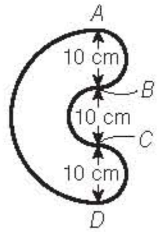
(a)



(b)

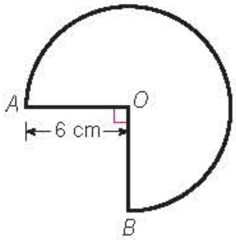


(c)

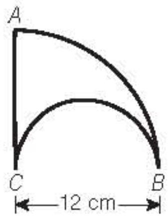


3. The following figures are formed by *quadrants*, semi-circles and straight lines, find the perimeter of each figure. (Express your answers in terms of π.)

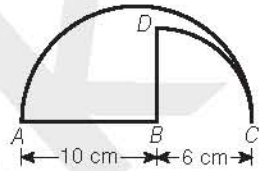
(a)



(b)



(c)



4. An iron wire is bent into a circle with the radius of 12 cm. What is the length of the iron wire?

5. The radii of two circles are 4 cm and 5 cm respectively. What is the difference between their circumferences? (Express your answer in terms of π.)

6. The tip of the minute-hand of a clock moves 94 cm in 1 hour. Find the length of the minute-hand.

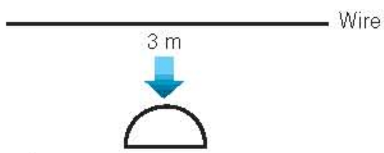
7. The diameter of a *discus* is 21.2 cm. Find the distance travelled by the discus in 10 revolutions.

8. A string of 44 cm long can wrap around a cylindrical *post* 10 times. What is the diameter of the post? (Take $\pi = \frac{22}{7}$.)

9. The radius of a car wheel is 0.4 m. How many revolutions does it make when the car travels 502.4 m? (Take $\pi = 3.14$.)

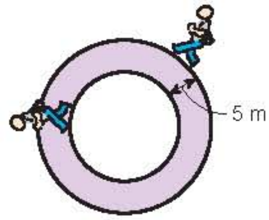
Level 2

10. A wire of 3 m long is bent to form a closed semi-circle. Find the radius of the semi-circle.



quadrant $\frac{1}{4}$個圓　discus 鐵餅　post 標竿

11. A circular track is 5 m wide. What is the difference in length between running a lap along the inner edge and the outer edge? (Express your answer in terms of π.)



12. Given that the difference between the circumferences of two circles is 6.28 cm, what is the difference between their radii? (Take $\pi = 3.14$.)

13. The ratio of the radii of two circles is $3:4$. What is the ratio of their circumferences?

14. The radius of a bicycle wheel is 0.35 m. If the wheel makes 2 revolutions per second, find the speed of the bicycle in m/s.

15. The diameters of the front and the ***rear*** wheels of a bicycle are 30 cm and 40 cm respectively. How many revolutions does the front wheel make when the rear wheel makes 24 revolutions?

16. The ***trunk*** of a tree in the shape of cylinder can be completely surrounded by 21 adults stretching their arms and holding hands. The width of an adult after stretching his/her arms is approximately 1.5 m. Estimate the diameter of the trunk, correct to the nearest m.

10.4 Areas of Circles

Class Activity 10.2

Aim: To explore the formula for the area of a circle

In the figure below, a circle is divided into 24 equal portions. They are rearranged into a figure looks like a parallelogram.

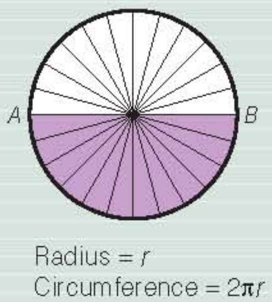


Figure I

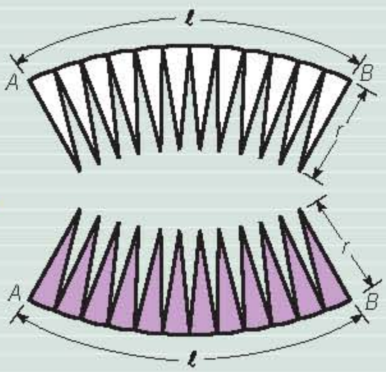


Figure II

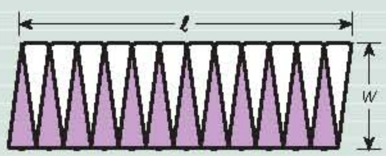


Figure III

rear 後方的　　trunk 樹幹

(a) Consider Figure I and Figure III. The value of w is ______________.

Show Ans.

(b) Consider Figure I and Figure III. The approximate value of ℓ is ______________.

Show Ans.

(c) Area of Figure III ≈ _______ × _______

Show Ans.

≈ _______ × _______

= _______

(d) What is the relation between the areas of Figure I and Figure III?

Show Ans.

____

(e) Hence, work out the area of the circle in Figure I. (Express your answer in terms of π and r.)

Show Ans.

____

In fact, the area A of a circle is πr^2, where r is the radius.

Student's CD

I.T. Tool

$$A = \pi r^2$$

Additional Example 10.11

Example 10.11 Finding the areas of circles with given radius/diameter

According to the given information, find the areas of the following circles. (Give your answers correct to the nearest 0.1 cm^2.)

(a) Radius = 4 cm

(b) Diameter = 12.5 cm

Solution

(a) Area of the circle $= \pi \times 4^2\text{ cm}^2$

$= \underline{\underline{50.3\text{ cm}^2}}$ (corr. to the nearest 0.1 cm^2)

(b) Radius of the circle $= \frac{1}{2} \times 12.5\text{ cm}$

$= 6.25\text{ cm}$

∴ Area of the circle $= \pi \times 6.25^2\text{ cm}^2$

$= \underline{\underline{122.7\text{ cm}^2}}$ (corr. to the nearest 0.1 cm^2)

Classwork 10.11

Solution

According to the given information, find the areas of the following circles. (Give your answers correct to the nearest 0.1 cm^2.)

(a) Radius = 8 cm

(b) Diameter = 27 cm

Additional Example 10.12

Example 10.12 Finding the diameter of a circle with given area

Find the diameter of the circular *coaster* if its area is 78.5 cm^2. (Take π = 3.14.)

Solution

Let r cm be the radius of the circular coaster.

$$3.14r^2 = 78.5$$
$$r^2 = 25$$
$$r = 5$$

$\therefore$ Diameter of the circular coaster $= 2r$ cm
$= \underline{\underline{10 \text{ cm}}}$

Classwork 10.12

Solution

According to the given information, find the diameters of the following circles.

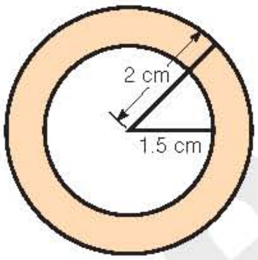
(a) Area = 379.94 cm^2 (Take π = 3.14.)

(b) Area = 154 cm^2 (Take $\pi = \frac{22}{7}$.)

Additional Example 10.13

Example 10.13 Finding the area of a circular ring

Find the area of the ring formed by two **concentric circles** with the radii of 1.5 cm and 2 cm. (Express your answer in terms of π.)



Solution

Area of the smaller circle $= \pi \times 1.5^2$ cm^2
$= 2.25\pi$ cm^2

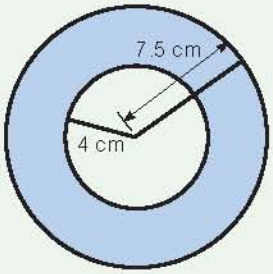
Area of the larger circle $= \pi \times 2^2$ cm^2
$= 4\pi$ cm^2

Area of the ring $= (4\pi - 2.25\pi)$ cm^2
$= \underline{\underline{1.75\pi \text{ cm}^2}}$

Classwork 10.13

Solution

In the figure, the radii of two concentric circles are 4 cm and 7.5 cm.



(a) Find the area of the shaded region.

(b) What is the difference between the circumferences of these two circles?

(Express your answers in terms of π.)

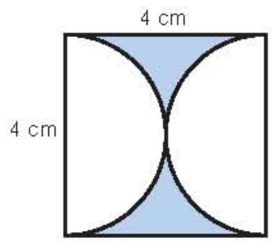
concentric circles 同心圓

coaster 杯墊

Skills Upgrading Corner 10.4

Solution

1. The figure is formed by semi-circles and a square. Find the area of the shaded region. (Give your answer correct to 1 decimal place.)

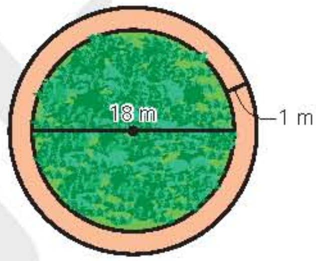


Solution

2. A wire of 30 cm long is bent to form a circle. Find the area of the circle. (Give your answer correct to 1 decimal place.)

Solution

3. A circular grassland with the diameter of 18 m is bounded by a path of 1 m wide. Find the area of the path. (Express your answer in terms of π.)



Exercise 10D

[In this exercise, give your answers correct to 1 decimal place if necessary.]

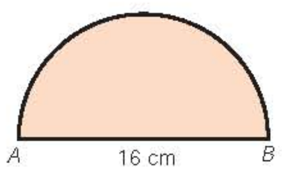
Level 1

1. Complete the following table. (Take $\pi = \frac{22}{7}$.)

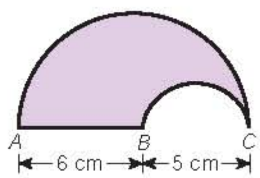
	Radius	Diameter	Area
(a)	14 cm		
(b)	7 cm		
(c)		21 cm	
(d)		11.2 cm	
(e)			962.5 cm^2
(f)			5 544 cm^2

2. The following figures are formed by semi-circles and straight lines, find the area of each figure. (Express your answers in terms of π.)

(a)

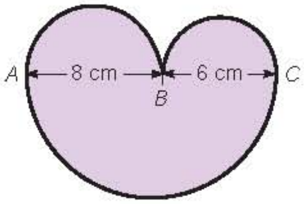


(b)

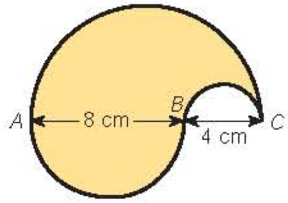


3. The following figures are formed by semi-circles, find the area of each figure. (Express your answers in terms of π.)

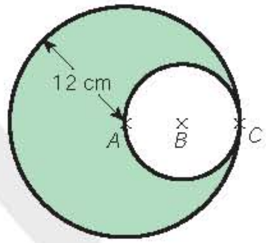
(a)



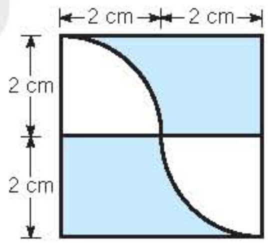
(b)



4. In the figure, A and B are the centres of the larger circle and the smaller circle respectively. The two circles touch at C. If the radius of the larger circle is 12 cm, find the area of the shaded region. (Express your answer in terms of π.)



5. The figure is formed by quadrants and a square. Find the area of the shaded region.



6. It is given that the diameter of a circular pool is 3.5 m. Find the area of the pool.

7. The area covered by the minute-hand of a clock in an hour is 200 cm^2. Find the length of the minute-hand.

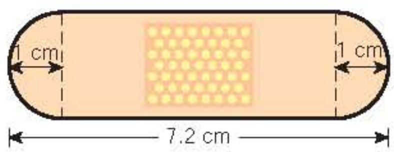
8. The area of a circular table is $20\ 096\text{ cm}^2$. Find the diameter of the circular table. (Take $\pi = 3.14$.)

Level 2

9. Find the ratio of the areas of two circles with radii 3 cm and 4 cm respectively.

10. Given that the ratio of the areas of two circles is 4 : 9, what is the ratio of their radii?

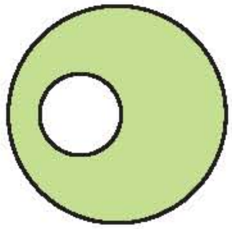
11. A piece of *adhesive bandage* is formed by a rectangle and two semi-circles at two ends as shown in the figure. Find its area.



adhesive bandage 藥水膠布

12. If the diameter of a circle doubles, how does its area change?

13. In the figure, the ratio of the diameters of the circles is 3 : 8.



(a) Find the ratio of the area of the smaller circle to that of the larger circle.

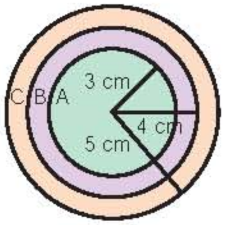
(b) If the area of the larger circle is 96 cm^2, find the area of the shaded region.

14. A rope of 376.8 cm long can wrap around a circle 10 times. Find the area of the circle. (Take $\pi = 3.14$.)

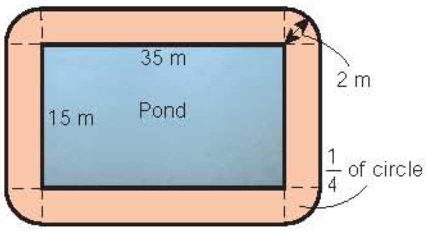
15. Given that the area of a circle is $2\ 464\text{ cm}^2$, find its circumference. (Take $\pi = \frac{22}{7}$.)

16. Given that the area of a circle is the sum of the areas of two circles with radii 5 cm and 12 cm respectively, find its radius.

17. The radii of the three concentric circles in the figure are 3 cm, 4 cm and 5 cm. Which region, A, B or C, has the smallest area?

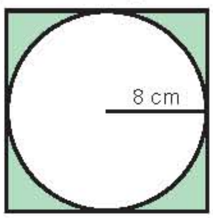


18. A rectangular pond with dimensions of $35\text{ m} \times 15\text{ m}$ is surrounded by a flower bed of 2 m wide. Find the area of the flower bed, correct to the nearest m^2.

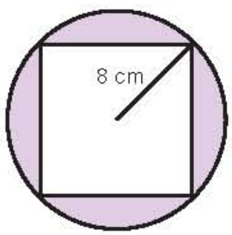


19. The following figures are formed by circles and squares. If the radii of circles in both figures are 8 cm, find the percentage of the area of the shaded region in each figure. (Give your answers correct to 3 significant figures if necessary.)

(a)



(b)



10.5 Lengths of Arcs and Areas of Sectors

In Figure 10.2, PQ is a part of the circumference. It is called an **arc** of the circle, and is denoted by 'arc PQ' or '$\overset{\frown}{PQ}$'. $\angle POQ$ is the angle subtended by arc PQ at centre O. It is called the **angle at the centre** subtended by arc PQ. The region enclosed by radii OP, OQ and the arc PQ is called a sector.

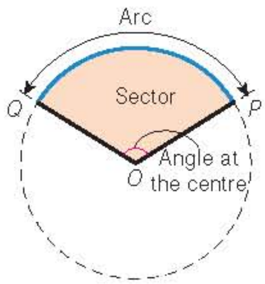


Figure 10.2

Class Activity 10.3

Aim: To explore the formulae for an arc length and the area of a sector

1. The circles in Figures I, II and III are of the same size. The shaded regions occupy $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of the circles in Figures I, II and III respectively.

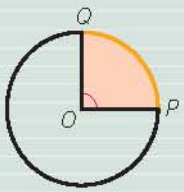


Figure I

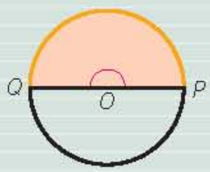


Figure II

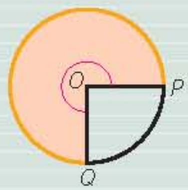


Figure III

(a) How does the length of $\overset{\frown}{PQ}$ change as the angle at the centre in the shaded region increases?

____

(b) Is the arc length proportional to the angle at the centre subtended by the arc?

☐ Yes ☐ No

(c) How does the area of the sector change as the angle at the centre in the shaded region increases?

____

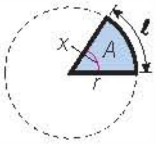
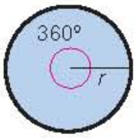
(d) Is the area of the sector proportional to the angle at the centre subtended by the arc?

☐ Yes ☐ No

arc 弧　　angle at the centre 圓心角

2. **(a)** Complete the following table. (Express your answers in terms of r.)

Show Ans.

Diagram	Area	Arc length	Angle at the centre
A sector 	A	ℓ	x
A circle 			360°

(b) Based on **2(a)** and the concept of ratios, fill in the blanks.

$$\frac{\text{Arc length}}{\text{Circumference}}=\frac{\text{Angle at the centre}}{\text{1 round angle}} \qquad \frac{\text{Area of a sector}}{\text{Area of a circle}}=\frac{\text{Angle at the centre}}{\text{1 round angle}}$$

$$\frac{\ell}{(\qquad)}=\frac{(\qquad)}{(\qquad)} \qquad \frac{A}{(\qquad)}=\frac{(\qquad)}{(\qquad)}$$

$$\ell=(\qquad)\times\frac{(\qquad)}{(\qquad)} \qquad A=(\qquad)\times\frac{(\qquad)}{(\qquad)}$$

According to the above Class Activity, we have the following formulae.

$$\text{Arc length}=2\pi r\times\frac{x}{360°}$$

$$\text{Area of sector}=\pi r^2\times\frac{x}{360°}$$

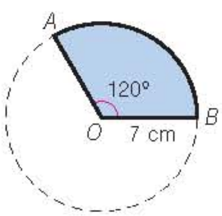
where x is the angle at the centre subtended by the arc.

I.T. Tool

Additional Example 10.14

Example 10.14 Finding the arc length and area of a sector

In the figure, find the length of $\overset{\frown}{AB}$ and the area of sector AOB. (Give your answers correct to 3 significant figures.)



Solution

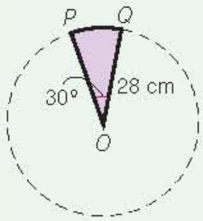
$$\text{Length of } \overset{\frown}{AB}=2\times\pi\times7\times\frac{120°}{360°}\text{ cm}$$
$$=\underline{14.7\text{ cm}}\text{ (corr. to 3 sig. fig.)}$$

$$\text{Area of sector } AOB=\pi\times7^2\times\frac{120°}{360°}\text{ cm}^2$$
$$=\underline{51.3\text{ cm}^2}\text{ (corr. to 3 sig. fig.)}$$

Classwork 10.14

Solution

In the figure, find the length of $\overset{\frown}{PQ}$ and the area of sector POQ. (Give your answers correct to 3 significant figures.)



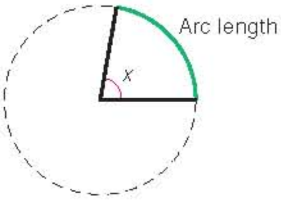
Additional Example 10.15

Example 10.15 Finding the angle at the centre

The arc length of a circle is $\frac{3}{8}$ of its circumference, find the angle at the centre subtended by the arc.

Solution

Let x be the angle at the centre subtended by the arc.



$$\frac{\text{Arc length}}{\text{Circumference}} = \frac{x}{360°}$$

$$\frac{3}{8} = \frac{x}{360°}$$

$$x = 360° \times \frac{3}{8}$$

$$= 135°$$

$\therefore$ The angle subtended by the arc is $135°$.

Classwork 10.15

Solution

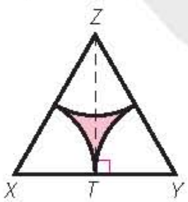
(a) The arc length of a circle is $\frac{5}{12}$ of its circumference. Find the angle at the centre subtended by the arc.

(b) The ratio of the arc length of a circle to its circumference is 7 : 15. Find the angle at the centre subtended by the arc.

Additional Example 10.16

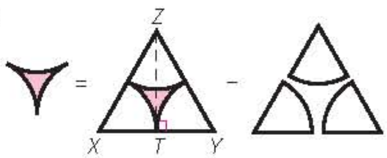
Example 10.16 Finding the area of a figure formed by arcs and straight lines

The figure shows an equilateral triangle XYZ with sides of 4 cm each. Find the area of the shaded region enclosed by 3 arcs with radii of 2 cm each. (Give your answer correct to 3 significant figures.)



Solution

[Analysis:



]

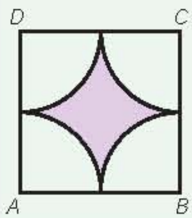
$\because$ XYZ is an equilateral triangle.

$\therefore$ $\angle XYZ = \angle YZX = \angle ZXY = 60°$

Classwork 10.16

Solution

The figure shows a square $ABCD$ with sides of 2 cm each. Find the area of the shaded region enclosed by 4 arcs with radii of 1 cm each. (Give your answer correct to 3 significant figures.)



In ΔXYZ,

sum of the areas of the three sectors

$= 3 \times \pi \times 2^2 \times \frac{60^\circ}{360^\circ}$ cm^2

$= 2\pi$ cm^2

$$\begin{aligned} ZT^2 + XT^2 &= XZ^2 \quad \text{(Pyth. theorem)} \\ ZT^2 &= XZ^2 - XT^2 \\ &= (4^2 - 2^2)\ \text{cm}^2 \\ &= 12\ \text{cm}^2 \end{aligned}$$

$\therefore \quad ZT = \sqrt{12}$ cm

Area of $\Delta XYZ = \frac{1}{2} \times 4 \times \sqrt{12}$ cm^2

$= 2\sqrt{12}$ cm^2

$\therefore$ Area of the shaded region

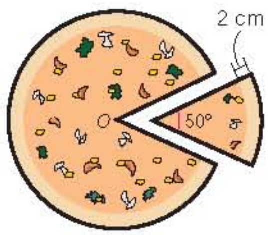
$= (2\sqrt{12} - 2\pi)$ cm^2

$= \underline{\underline{0.645\ \text{cm}^2}}$ (corr. to 3 sig. fig.)

Skills Upgrading Corner 10.5

Solution

1. A sector is cut from a circular pizza with the diameter of 30 cm and the *crust* of 2 cm wide as shown in the figure. Find the area of the crust of the sector. (Give your answer correct to 3 significant figures.)



Solution

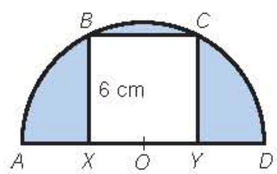
2. In the figure, $ABCD$ is a semi-circle, and $BCYX$ is a square with sides of 6 cm each.

(a) Find the length of OX.

(b) Find the radius of the semi-circle.

(c) Find the area of the shaded region.

(Give your answers correct to 3 significant figures if necessary.)



Exercise 10E

[In this exercise, give your answers correct to 3 significant figures if necessary.]

Level 1

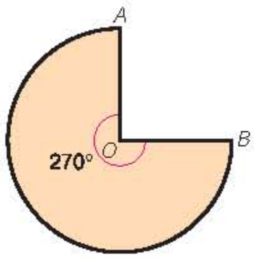
1. Complete the following table. (Express the arc lengths and areas of sectors in terms of π.)

	Radius	Angle at the centre	Arc length	Area of sector
(a)	4 cm	$135°$		
(b)	12 cm	$210°$		
(c)	6 cm		6π cm	
(d)		$45°$	8π cm	
(e)	8 cm			48π cm^2
(f)		$300°$		$1\,080\pi$ cm^2

2. An arc AB of a circle with the diameter of 12 cm subtends an angle of $54°$ at the centre. Find the length of arc AB.

3. An arc XY of a circle with the radius of 15 cm subtends an angle of $60°$ at the centre O. Find the area of sector XOY.

4. In the figure, the area of sector OAB is 108π cm^2, find the length of OA.



5. The arc length of a circle is $\frac{7}{10}$ of its circumference. Find the angle at the centre subtended by the arc.

6. The ratio of areas of a sector and the whole circle is $5 : 9$. Find the angle at the centre of the sector.

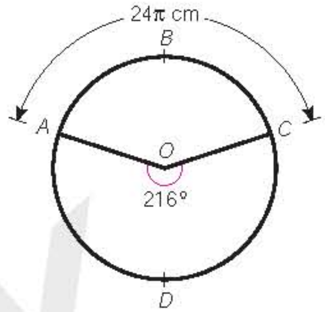
7. Given that the ratio of the arc lengths of two similar sectors is $4 : 5$, find the ratio of the areas of these two sectors.

8. The minute-hand of a clock is 10 cm long. How far does its tip move in 25 minutes?

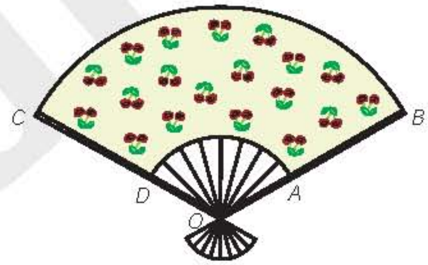
9. A wire forms an arc of a circle with the radius of 8.4 cm which subtends an angle of 30° at the centre. If the wire is bent into a circle, find the radius of the circle.

Level 2

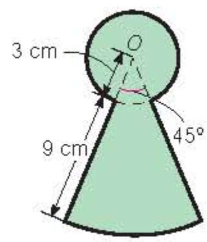
10. In the figure, the length of arc ABC is 24π cm, and arc ADC subtends an angle of 216° at the centre. Find the radius of the circle.



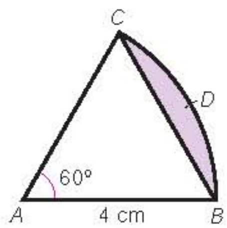
11. The figure shows a paper fan, where $OA = OD = 10$ cm, $AB = DC = 16$ cm and $\angle AOD = 120°$. Find the area of $ABCD$.



12. The figure is formed by a circle and a sector with the same centre O. Find the area of the figure.



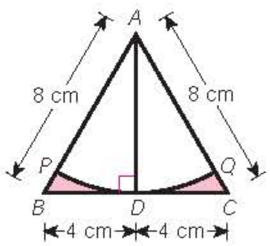
13. In the figure, ABC is an equilateral triangle with sides of 4 cm each and BDC is an arc with centre A. Find the shaded area.



14. In the figure, sector $APDQ$ is located in equilateral triangle ABC, where $AD \perp BC$, $AB = BC = CA = 8$ cm.

(a) Find the length of AD.

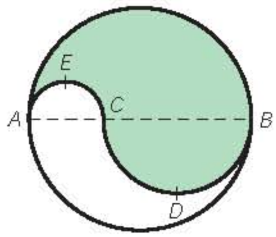
(b) Find the shaded area.



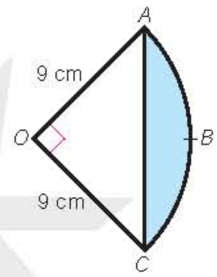
15. In the figure, ACB is the diameter of the circle where $BC = 2AC$. The area of the circle is x cm^2.

(a) Express the areas of the semi-circles BCD and ACE in terms of x.

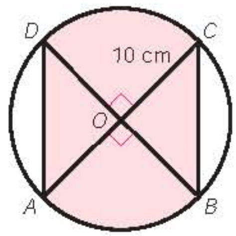
(b) If the area of the shaded region is 45 cm^2, find the value of x.



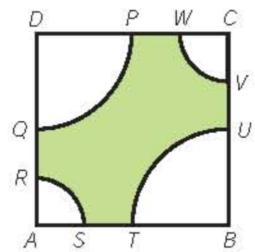
16. In the figure, AOC is a right-angled triangle, ABC is an arc with centre O. Find the perimeter of the shaded region.



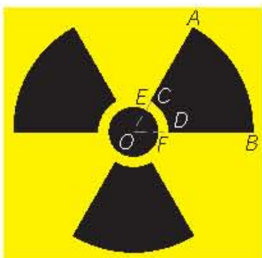
17. The figure shows a circle with the radius of 10 cm and with centre O. If $AOC \perp BOD$, find the perimeter of the shaded region.



18. In the figure, $ABCD$ is a square with sides of 8 cm each. If DPQ and BTU are sectors with radii of 4 cm each, and ARS and CVW are sectors with radii of 2 cm each, find the perimeter of the shaded region.



19. The figure shows a symbol representing radiation. It is formed by a circle and 3 equal parts. It is known that O is the centre of $\overset{\frown}{AB}$, $\overset{\frown}{CD}$ and the circle, $AC = 3.5$ cm, $CE = 0.5$ cm, $EO = 1$ cm and $\angle AOB = 60°$. Find the area of the regions in black.



10.6 Prisms and Cylinders

A Volumes of prisms and cylinders

I. Cuboids

In primary school, we have learned the formula for the volume of a cuboid.

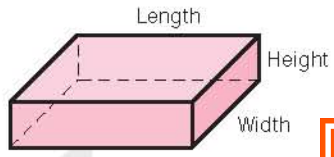


Figure 10.3

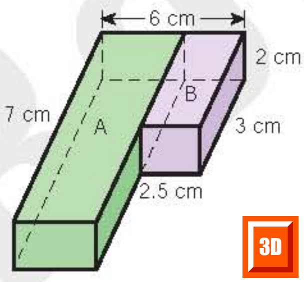
> Volume of cuboid = Length × Width × Height

Note: If the length, width and height of a cuboid are the same, it becomes a cube. Therefore, the volume of a cube is the cube of the length of a side.

Additional Example 10.17

Example 10.17 Finding the volume of cuboids

The solid shown is formed by two cuboids. Find its volume.



Solution

Volume of cuboid A $= 7\times(6-2.5)\times 2 \text{ cm}^3$

$= 49 \text{ cm}^3$

Volume of cuboid B $= 3\times 2.5\times 2 \text{ cm}^3$

$= 15 \text{ cm}^3$

$\therefore$ Volume of the solid $= (49+15) \text{ cm}^3$

$= \underline{\underline{64 \text{ cm}^3}}$

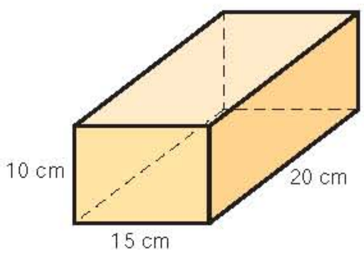
Classwork 10.17 **Solution**

The length, width and height of cuboid A are 24 cm, 12 cm and 12 cm respectively. For cuboid B, each of its sides is shorter than that of cuboid A by 4 cm. Find the total volume of the two cuboids.

Additional Example 10.18

Example 10.18 Problems involving cuboids and cubes

How many cubes with sides of 5 cm each are required to form the cuboid with dimensions of 20 cm × 15 cm × 10 cm?



Classwork 10.18 **Solution**

How many cubes with sides of 3 cm each are required to form a cuboid with its length of 21 cm, width of 15 cm and height of 12 cm?

Solution

The length of the cuboid equals 4 (i.e. $\frac{20}{5}$) times that of the cube.

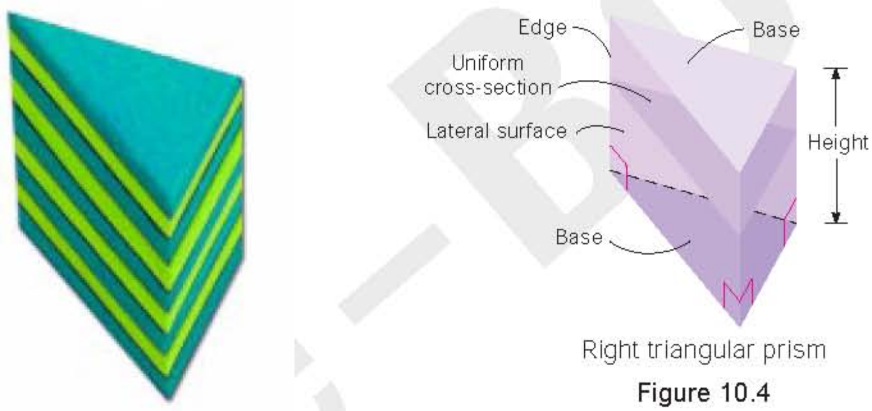
The width of the cuboid equals 3 (i.e. $\frac{15}{5}$) times that of the cube.

The height of the cuboid equals 2 (i.e. $\frac{10}{5}$) times that of the cube.

$$\begin{aligned} \therefore \quad \text{Number of cubes required} &= 4\times 3\times 2 \\ &= \underline{\underline{24}} \end{aligned}$$

II. Prisms

In Figure 10.4, when many identical triangular plates are piled up, a triangular prism is formed. The two ends of the prism are the bases. Each triangle parallel to the bases is called a uniform cross-section of the prism. The vertical distance between the bases is the **height** of the prism. The surfaces other than the two bases are called the **lateral surfaces**. The common side of two adjacent lateral surfaces is called the **edge**.



Right triangular prism

Figure 10.4

In conclusion, a prism is a solid with uniform cross-sections and its base is a polygon. If all the lateral surfaces of a prism are rectangles, i.e. each edge is perpendicular to the bases, it is called a **right prism**. Figure 10.5 shows some examples of right prisms.

Cuboid (a) | Right hexagonal prism (b)

Figure 10.5

height 高 lateral surface 側面 edge 棱 right prism 直立角柱體

Two identical right triangular prisms can form a cuboid. (See Figure 10.6)

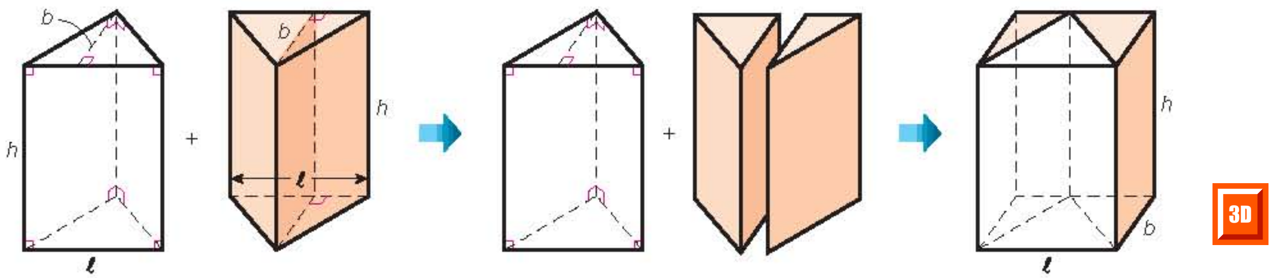


Figure 10.6

$$\therefore \quad \text{Volume of a right triangular prism} = \frac{1}{2} \times \text{Volume of a cuboid}$$
$$= \frac{1}{2} \times \ell \times b \times h$$
$$= (\frac{1}{2} \times \ell \times b) \times h$$
$$= \text{Base area} \times \text{Height}$$

If the triangular plates in Figure 10.4 are pushed to form an *oblique* triangular prism as shown in Figure 10.7, we can see that their heights and volumes are the same.

Oblique triangular prism

Figure 10.7

Therefore, we can use the formula below to find the volumes of the right triangular prism and oblique triangular prism.

Volume of triangular prism = Base area × Height

Since a polygon can be divided into triangles, a right prism can also be divided into many right triangular prisms (see Figure 10.8). Therefore,

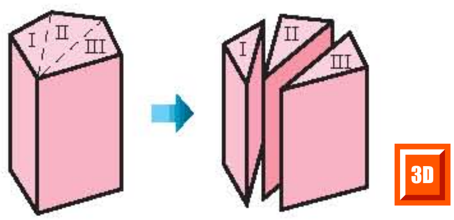


Figure 10.8

Volume of prism = Base area × Height

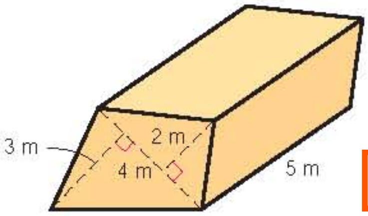
Note: Unless otherwise stated, prisms considered in this chapter are right prisms.

oblique 斜的

Additional Example 10.19

Example 10.19 Finding the volume of a prism with quadrilateral bases

In the figure, find the volume of the prism.



3D

Solution

$$\text{Base area} = \left(\frac{1}{2}\times 4\times 2+\frac{1}{2}\times 4\times 3\right)\text{ m}^2$$
$$= 10\text{ m}^2$$

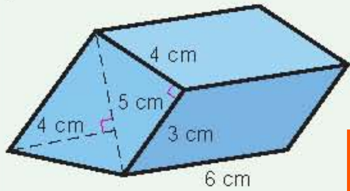
$\therefore$ Volume of the prism $= 10\times 5\text{ m}^3$

$= \underline{\underline{50\text{ m}^3}}$

Classwork 10.19

Solution

In the figure, find the volume of the prism.

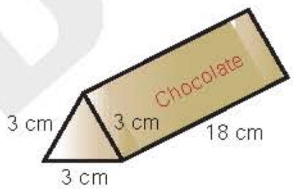


3D

Additional Example 10.20

Example 10.20 Finding the volume of a triangular prism

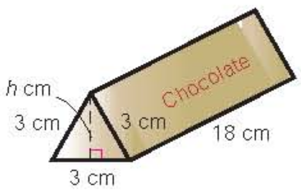
The base of a triangular chocolate bar is an equilateral triangle with sides of 3 cm each. If the chocolate bar is 18 cm long, find its volume. (Give your answer correct to 3 significant figures.)



3D

Solution

Let h cm be the height of the equilateral triangular base.



$h^2+\left(\frac{3}{2}\right)^2=3^2$ (Pyth. theorem)

$h^2=3^2-1.5^2$

$h=\sqrt{6.75}$

Base area of the triangular prism $=\frac{1}{2}\times 3\times\sqrt{6.75}\text{ cm}^2$

$=\frac{3}{2}\sqrt{6.75}\text{ cm}^2$

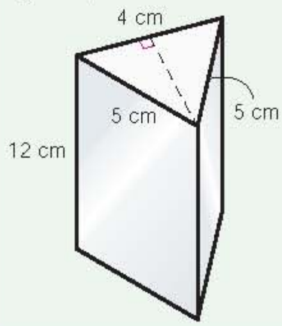
$\therefore$ Volume of the chocolate bar $=\frac{3}{2}\sqrt{6.75}\times 18\text{ cm}^3$

$=\underline{\underline{70.1\text{ cm}^3}}$ (corr. to 3 sig. fig.)

Classwork 10.20

Solution

The base of a crystal prism is an isosceles triangle with sides of 5 cm, 5 cm and 4 cm. If the crystal prism is 12 cm long, find its volume. (Give your answer correct to 3 significant figures.)



3D

III. Cylinders

A cylinder is a solid with circular bases and uniform cross-sections. (See Figure 10.9)

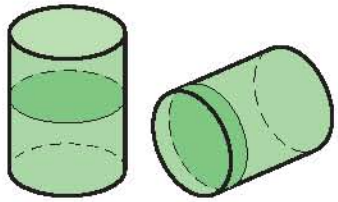


Figure 10.9

A cylinder can be divided into a number of solids look like triangular prisms as shown in Figure 10.10.

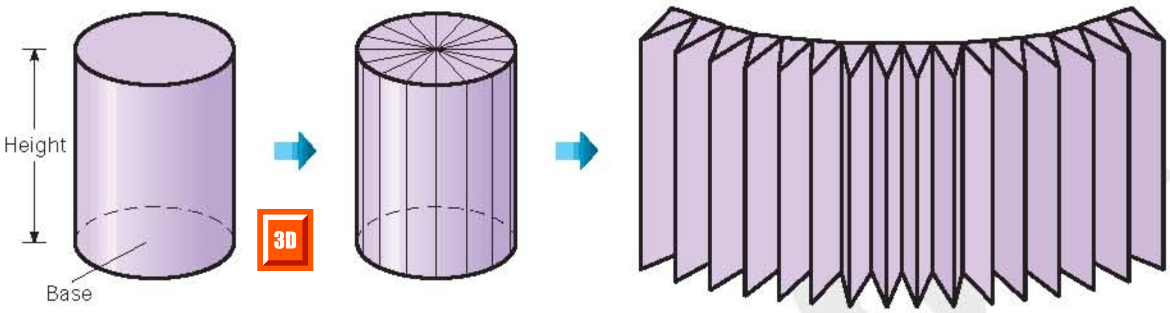


Figure 10.10

Therefore, it is not difficult to see that

'Volume of cylinder = Base area × Height', i.e.

$$\text{Volume of cylinder} = \pi r^2 h$$

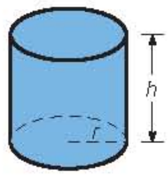


Figure 10.11

where r and h denote the base radius and the height of the cylinder respectively.

Notes: **(a)** For a right cylinder, the line joining the centres of the circular bases is perpendicular to the radii of the two bases. (See Figure 10.12)

(b) Unless otherwise stated, cylinders considered in this chapter are right cylinders.

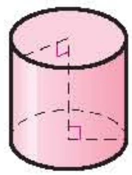
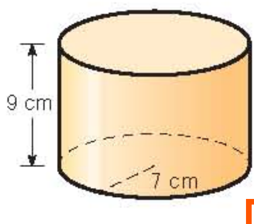


Figure 10.12

Additional Example 10.21

Example 10.21 Finding the volume of a cylinder

The base radius and the height of a cylinder are 7 cm and 9 cm respectively. Find its volume. (Express your answer in terms of π.)



Solution

$$\text{Volume of cylinder} = \pi \times 7^2 \times 9 \text{ cm}^3$$
$$= \underline{\underline{441\pi \text{ cm}^3}}$$

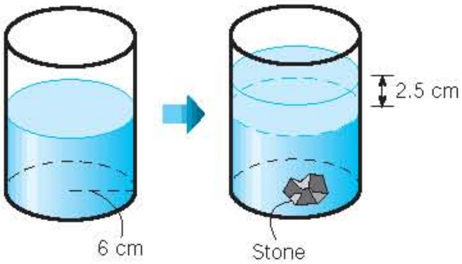
Classwork 10.21 Solution

The base diameter and the thickness of a \$5 coin are 2.7 cm and 0.3 cm respectively. Find its volume. (Give your answer correct to 2 significant figures.)

Additional Example 10.22

Example 10.22 Problems involving the volume of a cylinder

A stone is sunk to the bottom of a cylinder filled with water. If the base radius of the cylinder is 6 cm and the water level rises by 2.5 cm, find the volume of the stone. (Express your answer in terms of π.)



3D

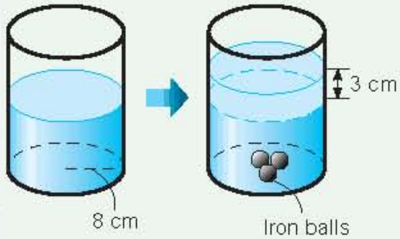
Solution

$$\begin{aligned}\text{Volume of the stone} &= \text{Volume of water risen}\\ &= \pi \times 6^2 \times 2.5 \text{ cm}^3\\ &= \underline{\underline{90\pi \text{ cm}^3}}\end{aligned}$$

Classwork 10.22

Solution

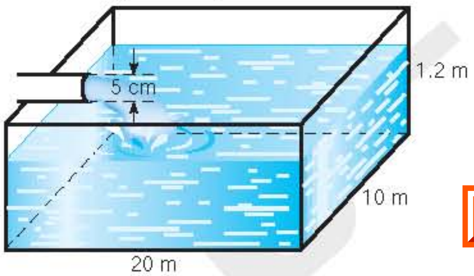
3 identical iron balls are sunk to the bottom of a cylinder filled with water. If the base radius of the cylinder is 8 cm and the water level rises by 3 cm, find the volume of each iron ball. (Express your answer in terms of π.)



Additional Example 10.23

Example 10.23 More complicated problems

The length, width and height of a rectangular tank are 20 m, 10 m and 1.2 m respectively.



3D

(a) Find the capacity of the tank.

(b) The tank is being filled with water by a pipe. The cross-section of the pipe is a circle with the diameter of 5 cm, and the water inside the pipe flows at 2 m/s. Find the volume of water in the tank after 1 hour. (Express your answer in terms of π.)

(c) Find the time required to fill up an empty tank by the pipe in **(b)**. (Give your answer correct to the nearest hour.)

Solution

(a) $$\begin{aligned}\text{Capacity of the tank} &= 20 \times 10 \times 1.2 \text{ m}^3\\ &= \underline{\underline{240 \text{ m}^3}}\end{aligned}$$

Classwork 10.23

Solution

The figure shows a mini water machine and a cylindrical cup. The base radius and the height of the cup are 3 cm and 6 cm respectively.

(a) Find the capacity of the cup. (Express your answer in terms of π.)

(b) It is known that the base radius of the cylindrical bottle on the top of the water machine is 10 cm. When water is released from the machine, the water level in the bottle falls at the speed of 5 mm/s. Find the time required to fill up the cup.

(b) [Analysis: The shape of water flowed out from the pipe each second can be regarded as a cylinder. Its base diameter and height are 5 cm and 2 m respectively.

]

Volume of water flowed out from the pipe each second

$= \pi \times 0.025^2 \times 2\ \text{m}^3$ ◂ Radius $= \frac{5}{2}$ cm $= 2.5$ cm $= 0.025$ m

$= 0.001\ 25\pi\ \text{m}^3$

$\therefore$ Volume of water in the tank after 1 hour

$= 60 \times 60 \times 0.001\ 25\pi\ \text{m}^3$ ◂ 1 hour = 60 minutes = 60 × 60 seconds

$= \underline{\underline{4.5\pi\ \text{m}^3}}$

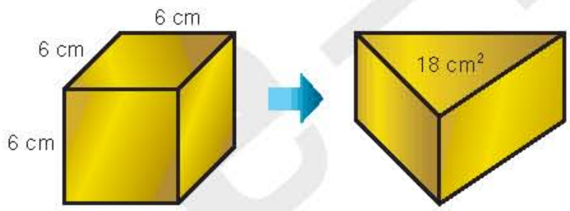
(c) Time required $= \frac{240}{4.5\pi}$ hours

$= \underline{\underline{17\text{ hours}}}$ (corr. to the nearest hour)

Skills Upgrading Corner 10.6

Solution

1. A solid metal cube with sides of 6 cm each is melted and recast into a right triangular prism with a base area of 18 cm^2. Find the height of the prism.



3D

Solution

2. 3 identical slices of lemon are sunk to the bottom of a cylindrical glass filled with water, and make the water level rise by 1 cm. If the base radius of the glass is 3 cm, find the volume of each slice of lemon. (Express your answer in terms of π.)

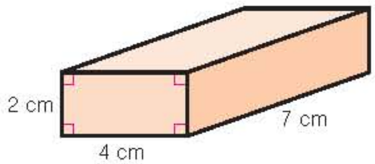
Exercise 10F

[In this exercise, give your answers correct to 3 significant figures if necessary.]

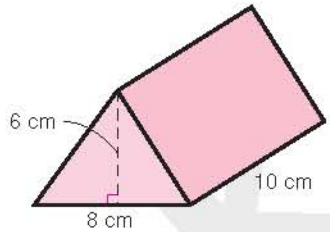
Level 1

1. Find the volumes of the following prisms.

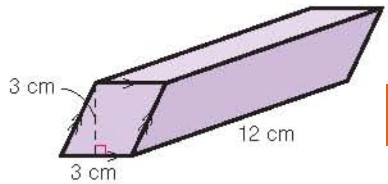
(a)



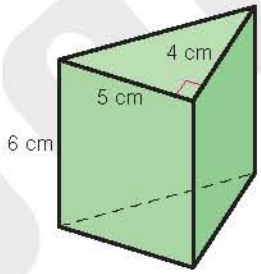
(b)



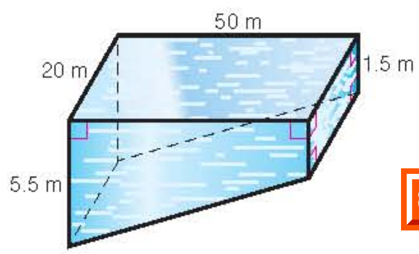
(c)



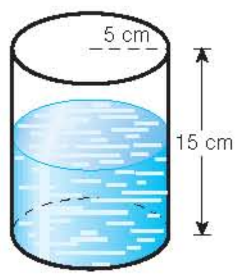
(d)



2. The figure shows a swimming pool. Find its capacity.



3. The figure shows a cylindrical glass which is half filled with water. Find the volume of water.

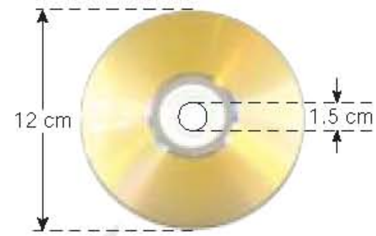


4. The dimensions of the base of a rectangular tank are 4 m $\times$ 5 m. If there is 250 m^3 of water in the tank, find the depth of water.

5. The depth of a cylindrical pond is 1.5 m and its capacity is 75 m^3. Find the base radius of the pond.

6. The base area and the height of a cylindrical container are 40 cm^2 and 8 cm respectively. Can it hold 355 cm^3 of juice?

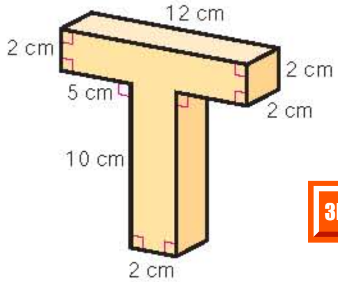
7. The outer and inner diameters of a compact disc are 12 cm and 1.5 cm respectively. If the thickness of the compact disc is 0.2 cm, find the volume of the compact disc.



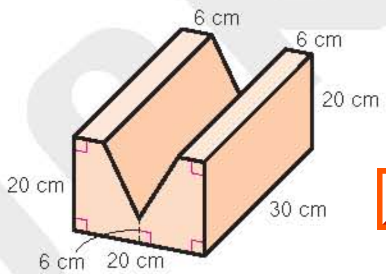
Level 2

8. Find the volume of each of the following prisms.

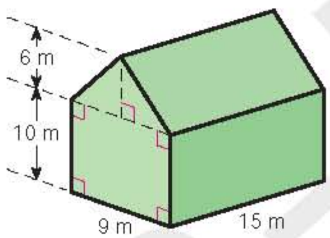
(a)



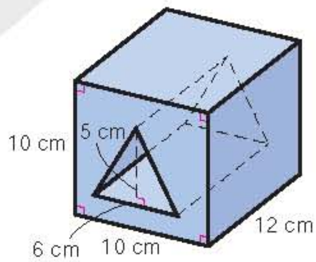
(b)



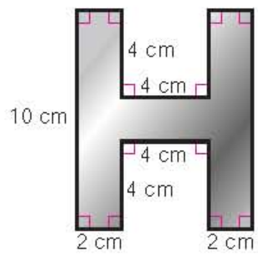
(c)



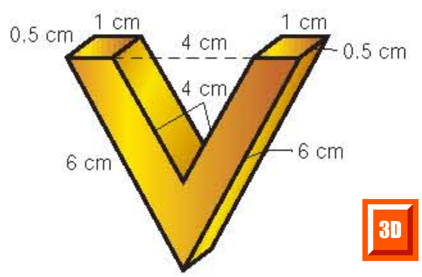
(d)



9. The figure shows the uniform cross-section of an H-shaped metal bar. If the bar is 2 m long, find its volume in cm^3.



10. The figure shows a metal prism. Find its volume.

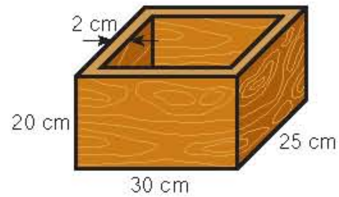


11. At most how many cuboids with the length of 5 cm, width of 3 cm and height of 2 cm can be cut from a cube with the sides of 30 cm each?

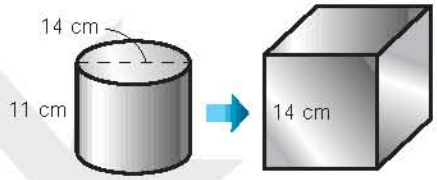
12. The figure shows a hollow cuboid box without cover. The box is made of wooden boards of 2 cm thick.

(a) Find the capacity of the box.

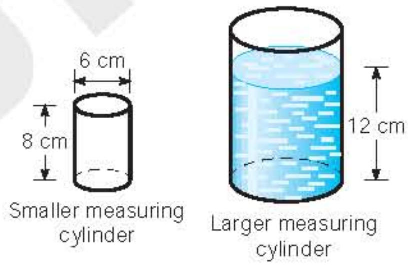
(b) Find the volume of wooden boards used.



13. A cylindrical lead bar with the base diameter of 14 cm and height of 11 cm is melted and recast into a cuboid with a square base and height of 14 cm. Find the length of the side of the base of the cuboid. (Take $\pi = \frac{22}{7}$.)



14. In the figure, the base diameter and the height of the smaller measuring cylinder are 6 cm and 8 cm respectively. After pouring water which equals to the capacity of 10 smaller measuring cylinders into the larger measuring cylinder, the water level of the larger measuring cylinder is 12 cm. Find the base diameter of the larger measuring cylinder.



15. **(a)** Figure I shows a cylindrical metal bar whose base diameter and height are 2 cm and 4 cm respectively. Find its volume. (Express your answer in terms of π.)

(b) Figure II shows a cylindrical jar whose base diameter and height are 12 cm and 25 cm respectively. The depth of water inside the jar is 20 cm. If 40 cylindrical metal bars in Figure I are put inside the jar, determine whether the water will overflow. Explain briefly.

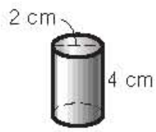


Figure I

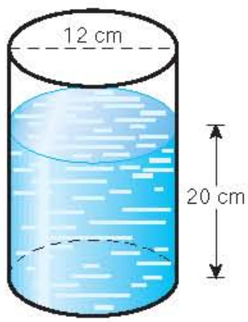
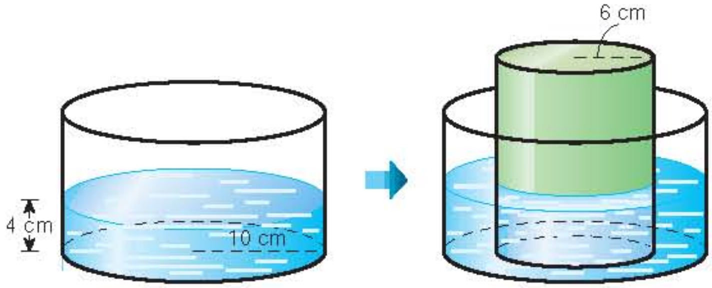


Figure II

16. The figure shows a cylindrical tank with the base radius of 10 cm with water inside. After putting a solid cylinder with the base radius of 6 cm into the water vertically, the water level rises. Find the rise in water level.



B Surface areas of prisms and cylinders

I. Prisms

To find the **surface area** of a prism, we can cut the prism along some edges and lay it flat as shown in Figure 10.13. Then we find the area of the plane surface.

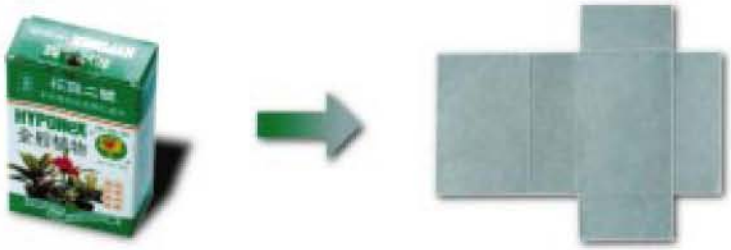


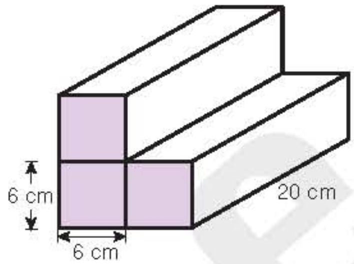
Figure 10.13

Thus, to find the total surface area of a prism, we need to find the area of its net.

Additional Example 10.24

Example 10.24 Finding the surface area of a prism

The prism below is made of three rectangular prisms whose lengths are 20 cm each and sides of square bases are 6 cm each. Find its total surface area.



Solution

Base area $= 3 \times (6 \times 6)\text{ cm}^2$

$= 108\text{ cm}^2$

Lateral surface area

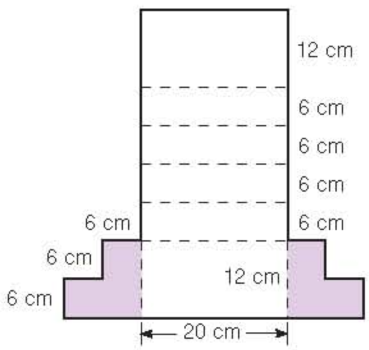
$= (12 + 6 + 6 + 6 + 6 + 12) \times 20\text{ cm}^2$

$= 960\text{ cm}^2$

$\therefore$ Total surface area of the prism

$= (2 \times 108 + 960)\text{ cm}^2$

$= \underline{\underline{1\ 176\text{ cm}^2}}$

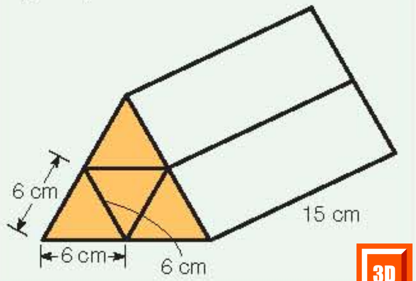


Note: In fact, lateral surface area of a prism = perimeter of the base × height.

Classwork 10.24

Solution

In the figure, the prism is made of four triangular prisms whose lengths are 15 cm each and sides of the equilateral triangular bases are 6 cm each. Find its total surface area. (Give your answer correct to 3 significant figures.)



surface area 表面面積

II. Cylinders

A rectangle can be rolled up to form the **curved surface** of a cylinder (see Figure 10.14). On the contrary, unrolling the curved surface of a cylinder gives a rectangle.

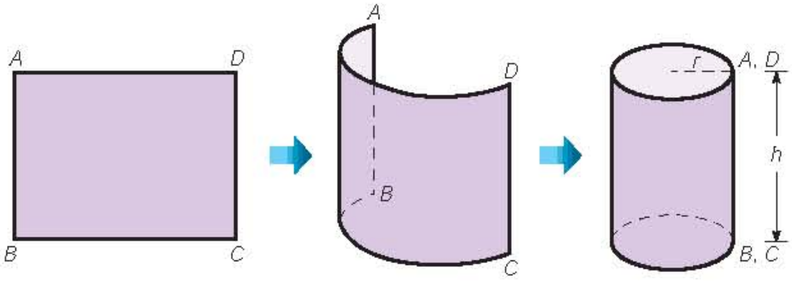


Figure 10.14

Thus, the area of the curved surface of the cylinder

= Area of rectangle $ABCD$

= $BC \times CD$

The length BC of the rectangle is the circumference of the base of the cylinder.

If the base radius of a cylinder is r and its height is h, then

Curved surface area of a cylinder $= 2\pi rh$

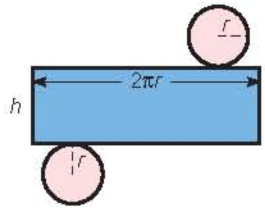


Figure 10.15

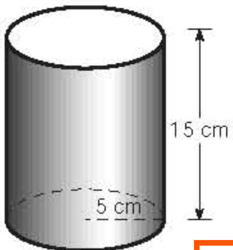
Therefore, including the two bases of the cylinder, we have:

Total surface area of a cylinder $= 2\pi rh + 2\pi r^2$

Additional Example 10.25

Example 10.25 Finding the total surface area of a cylinder

The base radius and the height of a hollow cylindrical tin can without a lid are 5 cm and 15 cm respectively. Find the area of the tin plate used in making the can. (Express your answer in terms of π.)



Solution

$\because$ The tin can has only one base.

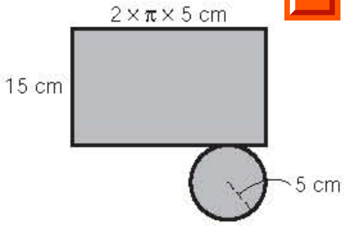
$\therefore$ Area of tin plate

= Base area + Curved surface area

$= (\pi \times 5^2 + 2 \times \pi \times 5 \times 15)\ \text{cm}^2$

$= (25\pi + 150\pi)\ \text{cm}^2$

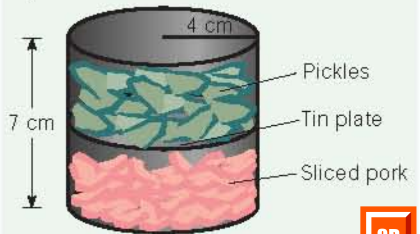
$= \underline{\underline{175\pi\ \text{cm}^2}}$



Classwork 10.25

Solution

A canned food, sliced pork with *pickles*, is packed inside a special cylindrical tin can. In the middle of the can, there is a tin plate separating the pickles and sliced pork. Find the area of the tin plate used in making the can. (Express your answer in terms of π.)

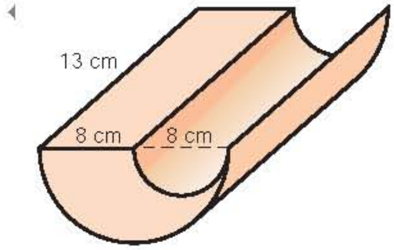


curved surface 曲面

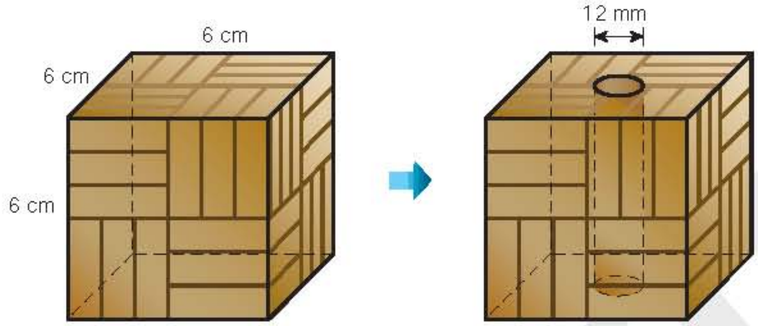
pickle 醃菜

Skills Upgrading Corner 10.7

1. In the figure, the uniform cross-sections of the solid are formed by semi-circles and straight lines. Find the total surface area of the solid. (Give your answer correct to 3 significant figures.)



2. In the figure, a cylindrical hole with the base diameter of 12 mm is drilled across a wooden cube with sides of 6 cm each. Find the change in the total surface area of the wooden cube. (Give your answer correct to 3 significant figures.)



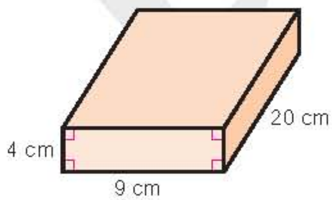
Exercise 10G

[In this exercise, give your answers correct to 3 significant figures if necessary.]

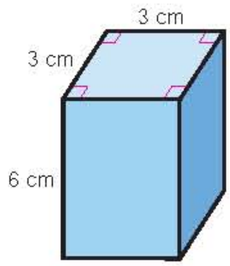
Level 1

The following solids have uniform cross-sections. Find the total surface area of each figure. (**1** – **5**)

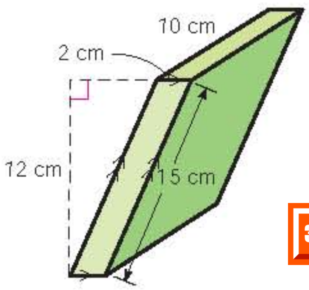
1. **(a)**



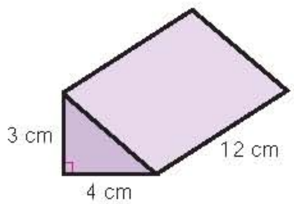
(b)



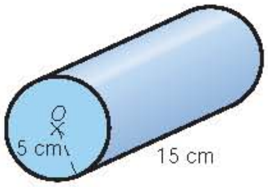
2. **(a)**



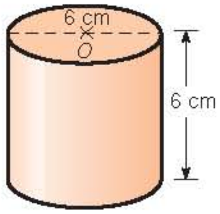
(b)



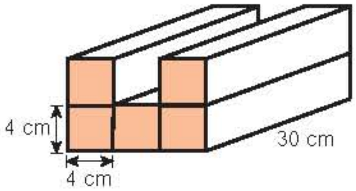
3. **(a)**



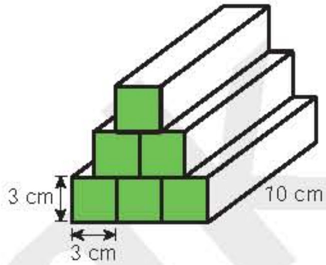
(b)



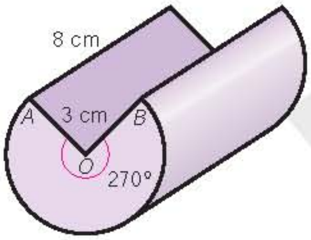
4. **(a)** The following solid is formed by 5 identical square prisms.



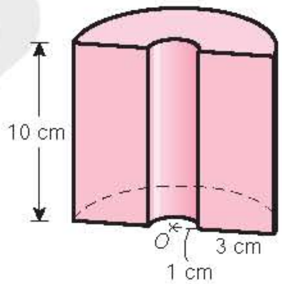
(b) The following solid is formed by 6 identical square prisms.



5. **(a)** The base of the solid is a sector with centre O.

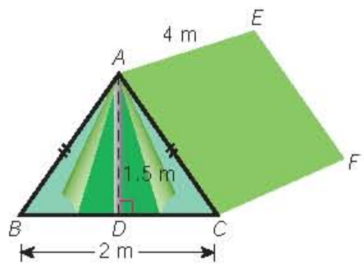


(b) The base of the solid is formed by 2 concentric semi-circles and straight lines, where O is the centre of the semi-circles.

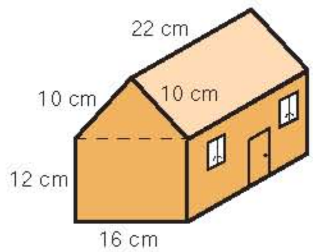


Level 2

6. The figure shows a tent in the shape of a triangular prism. Its uniform cross-section is an isosceles triangle with the base of 2 m and height of 1.5 m. The length of the tent is 4 m. Find the area of the material required to make this tent (including the base).



7. The figure shows a house model formed by a triangular prism and a cuboid. Find its total surface area (including the door and windows).



8. **(a)** A company has designed a magazine holder as shown in Figure I. Find the area of the cardboard required for making this magazine holder.

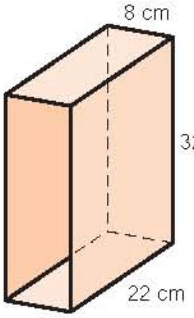


Figure I

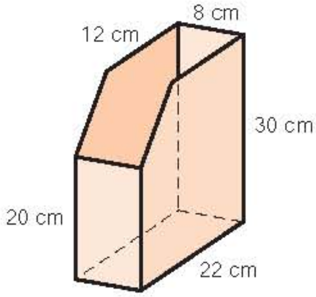


Figure II

(b) This company is considering another design of magazine holder as shown in Figure II. Find the area of cardboard required for making this new magazine holder.

(c) Which design uses less cardboard? How much cardboard can it save?

9. Figure I shows the original design of lump sugar in a large cuboid. Figure II shows the new design of lump sugar in a small cube.

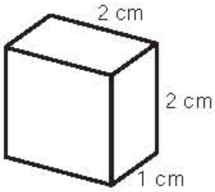


Figure I

Figure II

(a) How many small cubes of lump sugar can be made from a large cuboid of lump sugar?

(b) After changing lump sugar in a large cuboid into lump sugar in small cubes of the same volume, what is the percentage increase in the total surface area?

(c) The larger is the surface area of a piece of lump sugar touching the water, the faster the sugar dissolves. If a large cuboid of lump sugar and the same volume of lump sugar in small cubes are put into two glasses containing the same volume of water at the same temperature, which kind of sugar will dissolve faster? Explain briefly.

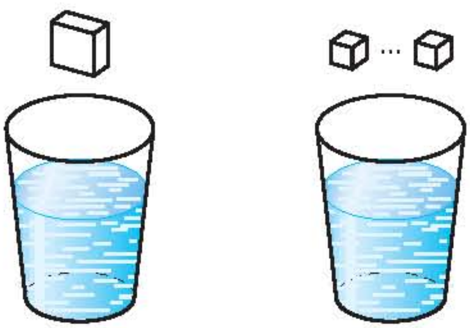


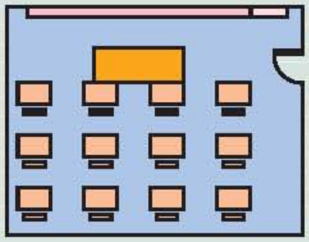
Figure III

10.7 Accumulation of Errors

Class Activity 10.4

Aim: To observe the accumulation of errors caused by using measured values with errors in formulae

Several students measured the length and width of their classroom and obtained the results of 7.4 m and 5.7 m respectively, correct to 2 significant figures.



(a) **(i)** Maximum absolute error of measurement = ________ m

Show Ans.

(ii) Complete the following table.

	Measured value	Lower limit	Upper limit
Length			
Width			

(b) Find the upper limit and lower limit of the area of classroom.

Show Ans.

Lower limit of the area of classroom = Lower limit of length × Lower limit of width

= ________ × ________ m^2

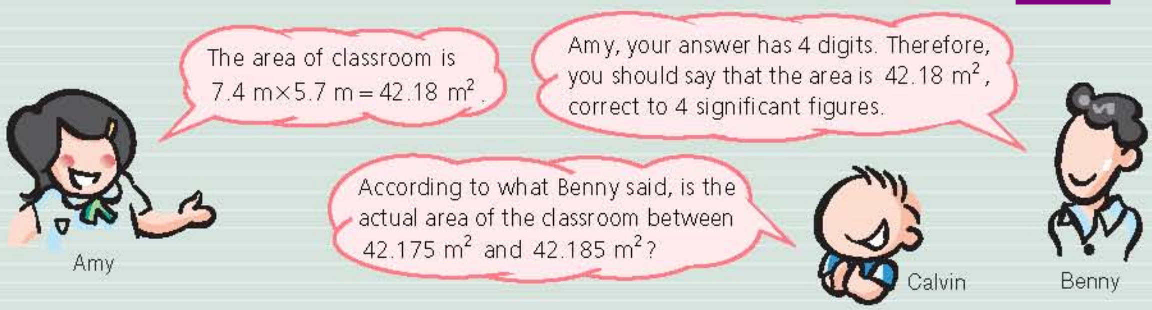
= ____________ m^2

Upper limit of the area of classroom = ________ × ________ m^2

= ____________ m^2

(c) Consider the conversation between the students, and comment on their views.

Show Ans.



____

____

____

Now I see ...

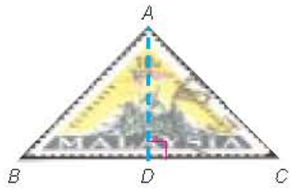
Using measured values with errors for calculation will end up with errors in the results.

From Class Activity 10.4, we have obtained the possible range of the actual area of the classroom through the consideration of the upper limits and lower limits of its length and width.

Additional Example 10.26

Example 10.26 Finding the possible range of area

The figure shows a triangular stamp ABC. By measurement, $AD = 2.4$ cm and $BC = 4.8$ cm, correct to 1 decimal place.



(a) **(i)** Find the upper limit and lower limit of AD.

(ii) Find the upper limit and lower limit of BC.

(b) Find the possible range of the area of the stamp.

Solution

[Analysis: Maximum absolute error of measurement $= 0.1 \times \frac{1}{2}$ cm $= 0.05$ cm.]

(a) **(i)** Upper limit of $AD = (2.4 + 0.05)$ cm $= \underline{\underline{2.45 \text{ cm}}}$

Lower limit of $AD = (2.4 - 0.05)$ cm $= \underline{\underline{2.35 \text{ cm}}}$

(ii) Upper limit of $BC = (4.8 + 0.05)$ cm $= \underline{\underline{4.85 \text{ cm}}}$

Lower limit of $BC = (4.8 - 0.05)$ cm $= \underline{\underline{4.75 \text{ cm}}}$

(b) Upper limit of the area of $\Delta ABC = \frac{1}{2} \times 2.45 \times 4.85 \text{ cm}^2$

$= 5.941\ 25 \text{ cm}^2$

Lower limit of the area of $\Delta ABC = \frac{1}{2} \times 2.35 \times 4.75 \text{ cm}^2$

$= 5.581\ 25 \text{ cm}^2$

$\therefore$ $\underline{\underline{5.581\ 25 \text{ cm}^2 \le \text{Area of the stamp} < 5.941\ 25 \text{ cm}^2}}$

Classwork 10.26

Solution

By measurement, the radius of a $10 coin is 1.2 cm, correct to 2 significant figures.

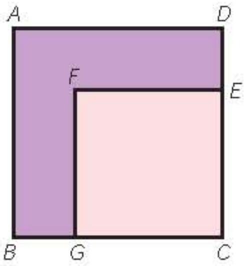
(a) Find the upper limit and lower limit of the radius of the coin.

(b) Find the possible range of the base area of the coin. (Express your answer in terms of π.)

Additional Example 10.27

Example 10.27 Finding the upper limit and lower limit of the difference of lengths

The figure shows squares $ABCD$ and $CEFG$. It is given that the lengths of AB and CE are 7.0 cm and 5.0 cm respectively, correct to 2 significant figures.



(a) Find the upper limit and lower limit of CD.

(b) Find the upper limit and lower limit of CE.

(c) Find the upper limit and lower limit of DE.

Solution

(a) Upper limit of $CD = \underline{\underline{7.05 \text{ cm}}}$

Lower limit of $CD = \underline{\underline{6.95 \text{ cm}}}$

(b) Upper limit of $CE = \underline{\underline{5.05 \text{ cm}}}$

Lower limit of $CE = \underline{\underline{4.95 \text{ cm}}}$

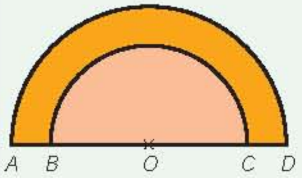
(c)
$$\begin{aligned}\text{Upper limit of } DE &= \text{Upper limit of } CD - \text{Lower limit of } CE\\ &= (7.05 - 4.95) \text{ cm}\\ &= \underline{\underline{2.1 \text{ cm}}}\end{aligned}$$

$$\begin{aligned}\text{Lower limit of } DE &= \text{Lower limit of } CD - \text{Upper limit of } CE\\ &= (6.95 - 5.05) \text{ cm}\\ &= \underline{\underline{1.9 \text{ cm}}}\end{aligned}$$

Classwork 10.27

Solution

The figure shows two concentric semi-circles with centre O, where $OC = 4.4$ cm and $OD = 6.6$ cm, correct to 1 decimal place.



(a) Find the upper limit and lower limit of OD.

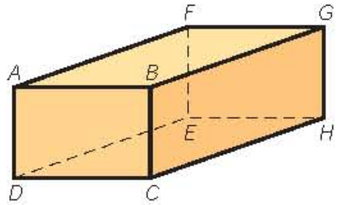
(b) Find the upper limit and lower limit of OC.

(c) Find the upper limit and lower limit of CD.

Skills Upgrading Corner 10.8

Solution

The figure shows a cuboid $ABCDEFGH$, where $AB = 1.8$ cm, $BC = 1.6$ cm and $AF = 2.8$ cm, correct to 1 decimal place.



3D

(a) **(i)** Find the upper limit and lower limit of AB.

(ii) Find the upper limit and lower limit of BC.

(iii) Find the upper limit and lower limit of AF.

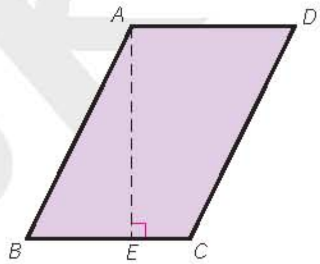
(b) Find the possible range of the volume of the cuboid.

Exercise 10H

Level 1

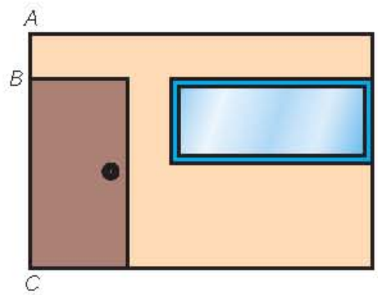
1. The length and width of a sheet of paper are 24 cm and 20 cm respectively, correct to 2 significant figures.
   (a) (i) Find the upper limit and lower limit of the length.
       (ii) Find the upper limit and lower limit of the width.
   (b) Find the possible range of the perimeter of the sheet of paper.

2. The figure shows a parallelogram, where $AE = 15.7$ cm and $BC = 12.8$ cm, correct to 3 significant figures.
   (a) (i) Find the upper limit and lower limit of AE.
       (ii) Find the upper limit and lower limit of BC.
   (b) Find the possible range of the area of the parallelogram.



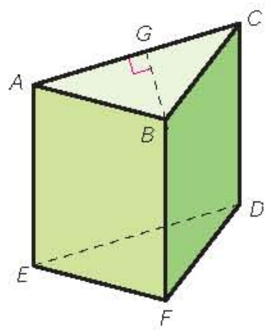
3. The base radius of a cylindrical glass is 7.42 cm, correct to 3 significant figures.
   (a) Find the upper limit and lower limit of the base radius.
   (b) Find, in terms of π, the possible range of the circumference of the base of the glass.

4. The figure shows a wall of a classroom. The lengths of AC and BC are 2.48 m and 2.00 m respectively, correct to 3 significant figures. Find the possible range of AB.



Level 2

5. The figure shows a triangular prism, where $AC = 4.8$ cm, $BG = 2.5$ cm and $AE = 9.5$ cm, correct to 1 decimal place.
   (a) (i) Find the upper limit and lower limit of AC.
       (ii) Find the upper limit and lower limit of BG.
       (iii) Find the upper limit and lower limit of AE.
   (b) Find the possible range of the volume of the triangular prism.



6. The base radius and the height of a cylindrical can are 4.1 cm and 6.3 cm respectively, correct to 2 significant figures.
   (a) **(i)** Find the upper limit and lower limit of the base radius.
   (ii) Find the upper limit and lower limit of the height.
   (b) Find, in terms of π, the possible range of the volume of the can.

Chapter Summary

Chapter Summary

A. Term Introduced

[This is a quiz to check your understanding of some special terms in this chapter. Match items in column A to column B appropriately.]

	Column A			Column B
1.	π	•	• (a)	Part of a circumference.
2.	Arc	•	• (b)	A solid with uniform cross-sections and circular bases.
3.	Sector	•	• (c)	A solid with uniform cross-sections and bases in polygons.
4.	Prism	•	• (d)	The ratio of the circumference of a circle to its diameter.
5.	Cylinder	•	• (e)	The region of a circle enclosed by two radii and an arc.

B. Fact to Remember

1. For any two similar figures, if ℓ_1 and ℓ_2 are the lengths of their corresponding sides while A_1 and A_2 are their areas, then $\frac{A_1}{A_2} = (\frac{\ell_1}{\ell_2})^2$.
2. Circumference $C = \pi d = 2\pi r$, where d is the diameter and r is the radius of the circle.
3. Area of circle $A = \pi r^2$, where r is the radius.
4. Arc length $= 2\pi r \times \frac{x}{360^\circ}$, where x is the angle at the centre subtended by the arc.
5. Area of sector $= \pi r^2 \times \frac{x}{360^\circ}$, where x is the angle at the centre subtended by the arc.
6. Volume of cuboid = Length × Width × Height
7. Volume of prism = Base area × Height

8. Volume of cylinder $= \pi r^2 h$, where r and h denote the base radius and the height of the cylinder respectively.

9. Curved surface area of a cylinder $= 2\pi rh$, where r and h denote the base radius and the height of the cylinder respectively.

10. Total surface area of a cylinder $= 2\pi rh + 2\pi r^2$, where r and h denote the base radius and the height of the cylinder respectively.

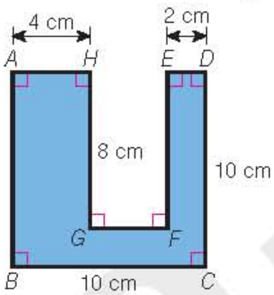
Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

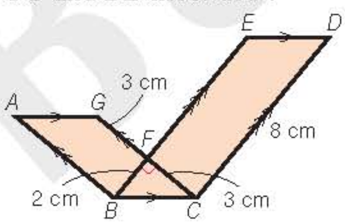
Section

1. Find the area of each figure. 10.1

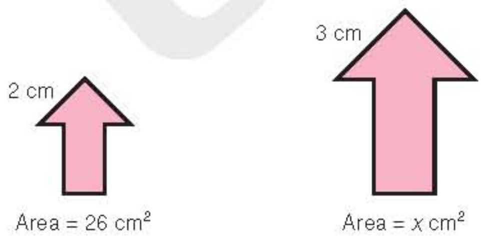
(a)



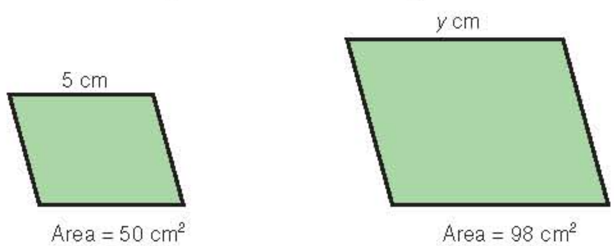
(b) GC and BE meet at F.



2. (a) The following are two similar figures. $x =$ __________. 10.2

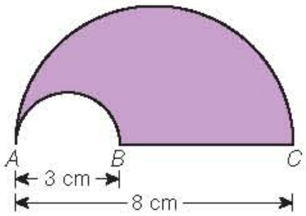


(b) The following are two similar figures. Find the value of y.



3. **(a)** It is known that the circumference of a circle is 66 cm. The radius of the circle is ________ cm. (Take $\pi = \frac{22}{7}$.) 10.3

(b) The figure is formed by semi-circles and a straight line. Find the perimeter of the figure. (Give your answer correct to 3 significant figures.)



4. **(a)** Given that the diameter of a circle is 6 cm, its area is ________ π cm^2. 10.4

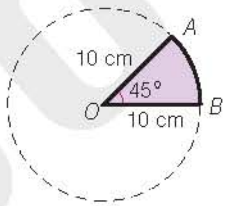
(b) Given that the circumference of a circle is 8π cm, find its area. (Express your answer in terms of π.)

5. In the figure, 10.5

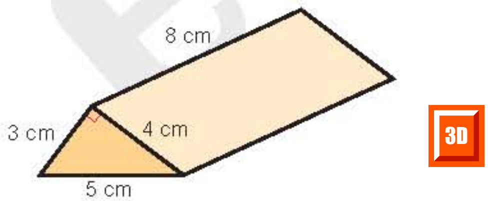
(a) the length of arc AB is ________ cm.

(b) find the area of sector AOB.

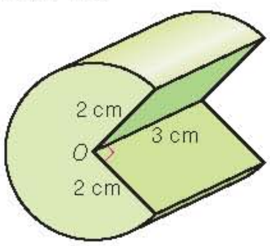
(Give your answers correct to 3 significant figures.)



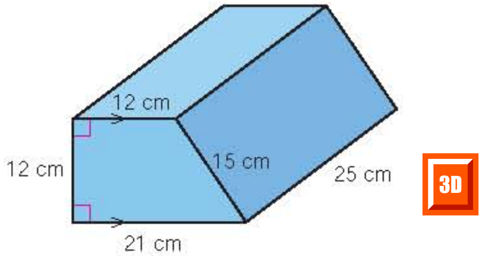
6. **(a)** In the figure, the base of a triangular prism is a right-angled triangle. The volume of the triangular prism is ________ cm^3. 10.6A



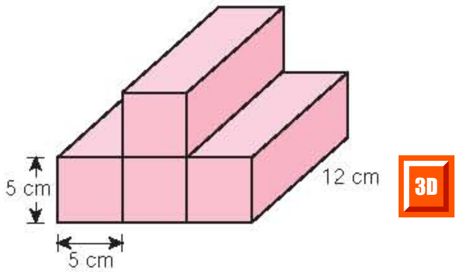
(b) Find the volume of the following solid with uniform cross-sections. (Give your answer correct to 3 significant figures.)



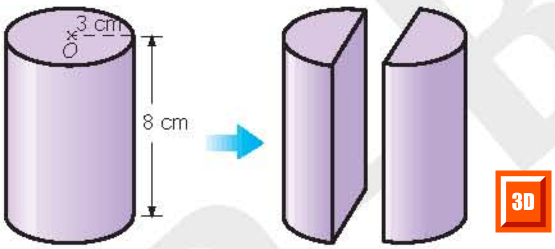
7. **(a)** In the figure, the total surface area of the prism is __________ cm^2. 10.6B



(b) The following prism is formed by 4 identical square prisms. Find its total surface area.



8. The figure shows a solid cylinder. 10.6B



(a) The total surface area of the cylinder is __________ π cm^2.

(b) If the cylinder is cut into two equal halves as shown, find the total surface area of these two parts. (Express your answer in terms of π.)

9. Each side of a cube is 14 cm long, correct to 2 significant figures. 10.7

(a) Find the upper limit and lower limit of the side.

(b) Find the possible range of the surface area of the cube.

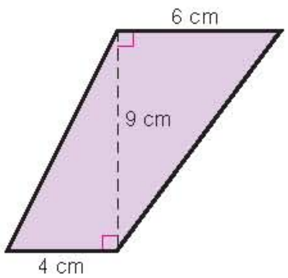
Revision Exercise 10

[In this exercise, give your answers correct to 3 significant figures if necessary.]

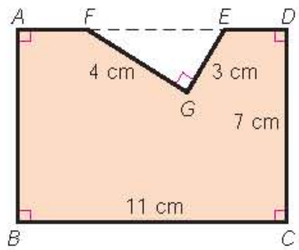
Level 1

1. Find the area of the shaded region in each of the following figures.

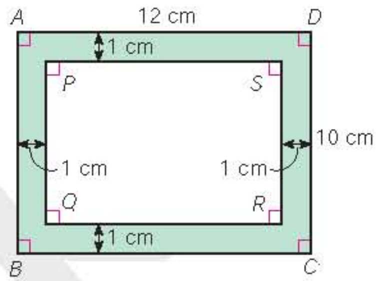
(a)



(b)

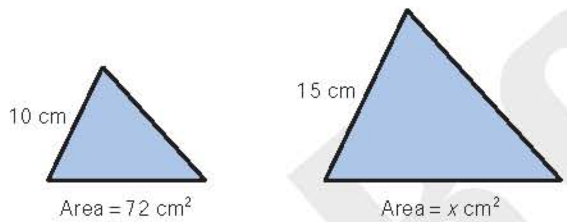


(c)

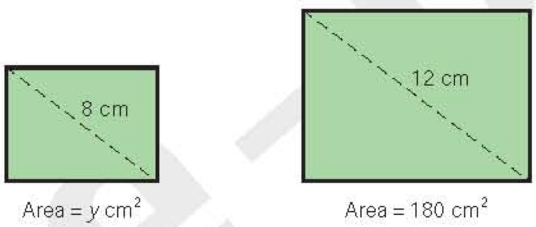


2. Find the unknown in each of the following pairs of similar figures.

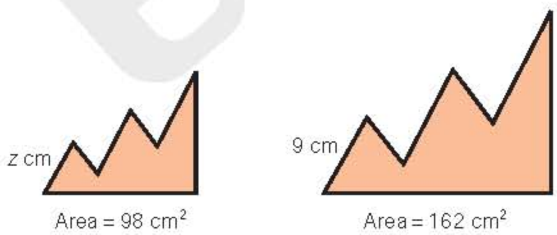
(a)



(b)

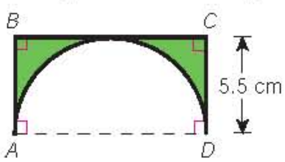


(c)

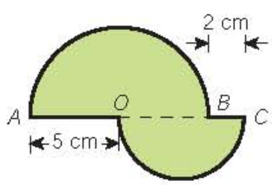


3. The following figures are formed by semi-circles and straight lines. Find the perimeter and area of the shaded region in each figure.

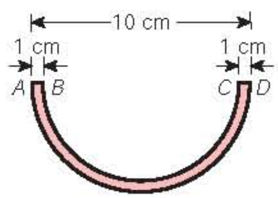
(a)



(b)

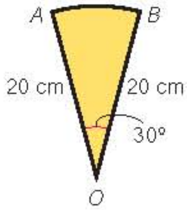


(c)

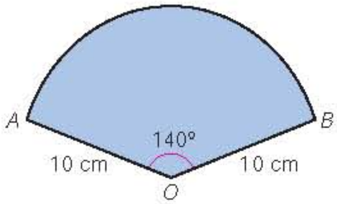


4. Find the perimeter and area of each of the following sectors.

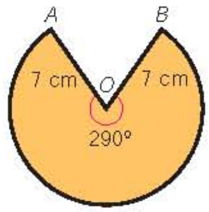
(a)



(b)

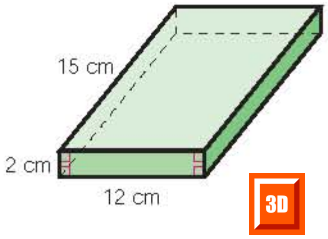


(c)

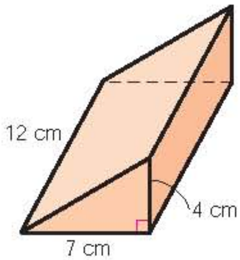


5. Find the volume and total surface area of each of the following prisms.

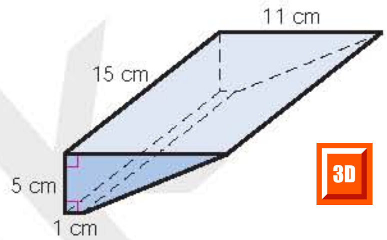
(a)



(b)



(c)



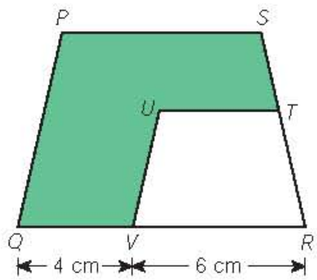
6. The base of an open-top cylindrical steel tank is 2 m in diameter and 3 m in height.

(a) Find the capacity of the tank.

(b) Find the area of steel plate required to make this tank.

(Express your answers in terms of π.)

7. In the figure, $PQRS$ and $UVRT$ are two similar trapeziums. If the area of $UVRT$ is 54 cm^2, find the area of the shaded region.



8. The uniform cross-section of a box consists of a rectangle and a semi-circle as shown in the figure.

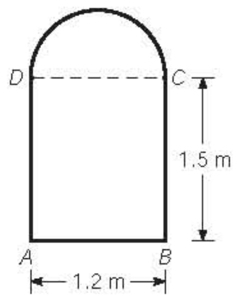
(a) Find the perimeter of the uniform cross-section.

(b) Find the area of the uniform cross-section.

(c) If the length of the box is 2.5 m,

(i) find the volume of the box.

(ii) find the total surface area of the box.

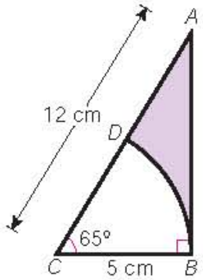


9. In the figure, ABC is a right-angled triangle, where $\angle ABC$ is a right angle. If an arc with radius BC and centre C cuts AC at D,

(a) find the length of AB.

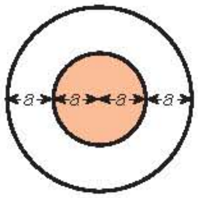
(b) find the area of sector DCB.

(c) find the area of the shaded region.

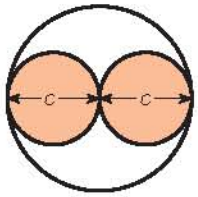


10. What percentage of the area of the largest circle is shaded in each of the following figures?

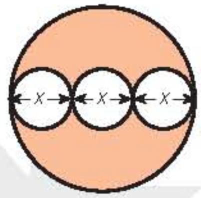
(a)



(b)

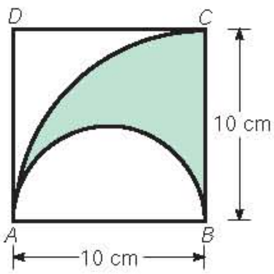


(c)

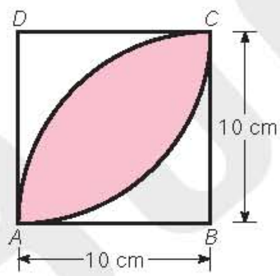


11. In each of the following figures, $ABCD$ is a square with sides of 10 cm each. The centres of the arcs are the vertices or the mid-points of the sides of the squares. Find the areas of the shaded regions.

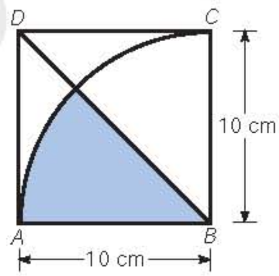
(a)



(b)



(c)

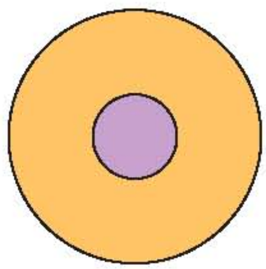


12. The figure shows two concentric circles whose diameters are 3.0 cm and 9.0 cm respectively, correct to 1 decimal place.

(a) Find the upper limit and lower limit of the diameter of the smaller circle.

(b) Find the upper limit and lower limit of the diameter of the larger circle.

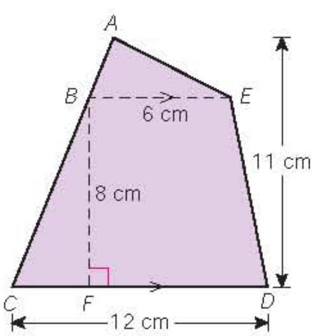
(c) Find the possible range of the circumference of each circle. (Express your answers in terms of π.)



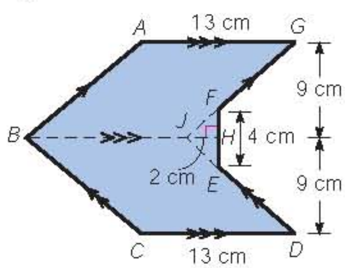
Level 2

13. Find the area of each of the following figures.

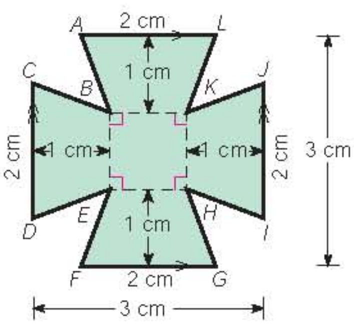
(a)



(b)

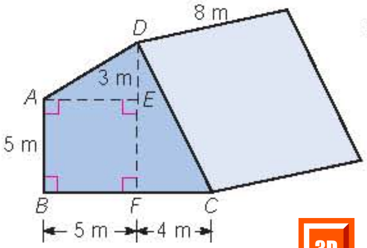


(c)

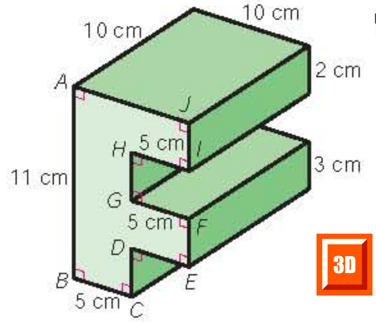


14. Find the volume of each of the following prisms.

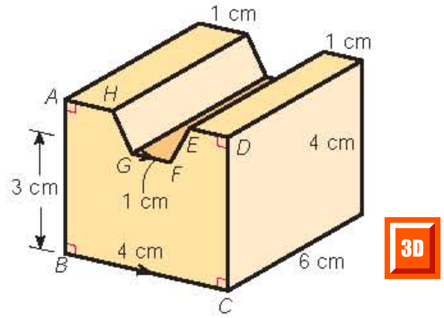
(a)



(b)



(c)

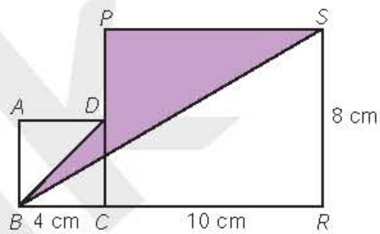


15. In the figure, $ABCD$ is a square with sides of 4 cm each and $PCRS$ is a rectangle with dimensions of 10 cm × 8 cm.

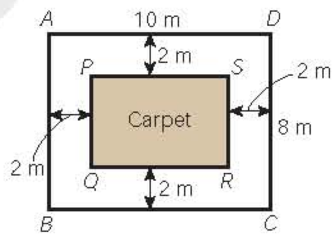
(a) Find the sum of the areas of square $ABCD$ and rectangle $PCRS$.

(b) Find the respective areas of ΔABD and ΔBRS.

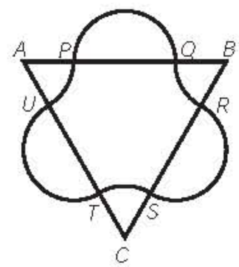
(c) Hence, find the area of the shaded region.



16. A rectangular carpet is put in the centre of a living room with dimensions of 10 m × 8 m. It leaves a passage of 2 m wide around the carpet. If the carpet costs \$120 per m^2, find the price of the carpet.



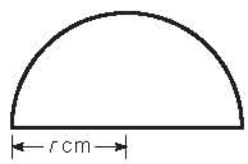
17. In the figure, ABC is an equilateral triangle with sides of 8 cm each. Find the length of the curve which is composed of arcs with radii of 2 cm each.



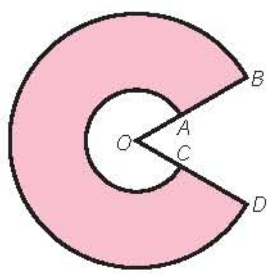
18. A wire of 50 cm long is bent to form a semi-circle with the radius of r cm.

(a) Express the perimeter of the semi-circle in terms of r and π.

(b) Find the value of r.



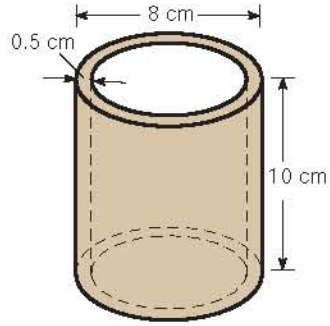
19. The figure shows two similar sectors whose radii are 6 cm and 15 cm respectively. If the area of the shaded region is 462 cm^2, find the areas of these two sectors.



20. The figure shows a cylindrical glass of uniform thickness. Its outer diameter is 8 cm, its height is 10 cm and its thickness is 0.5 cm.

(a) Find its capacity.

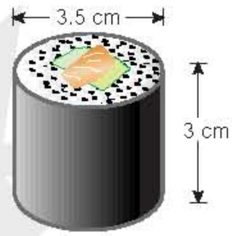
(b) Find the volume of material required for making the glass.



21. The figure shows a piece of *sushi roll* in the shape of cylinder. The sushi roll is made of rice, *cucumber*, *salmon* and *avocado* wrapping in two layers of *seaweed*. The cucumber, salmon and avocado are cut in the shape of prism with base areas of 0.25 cm^2, 0.3 cm^2 and 0.2 cm^2 respectively. It is given that the diameter of the sushi roll is 3.5 cm and its height is 3 cm.

(a) Find the total area of the seaweed sheets used.

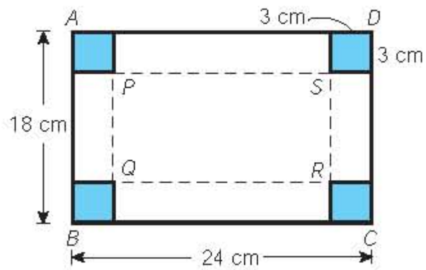
(b) If there are two strips of cucumber, a slice of salmon and a strip of avocado in the sushi, find the volume of the rice in the sushi.



22. The figure shows a rectangular cardboard $ABCD$ with dimensions of 24 cm × 18 cm. A square with sides of 3 cm each is cut from each corner of the cardboard. If the cardboard is folded along the dotted lines to form a rectangular container,

(a) find the lengths of PS and PQ.

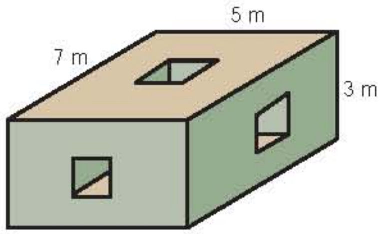
(b) find the capacity of the container.



23. The figure shows a cuboid with dimensions of 7 m × 5 m × 3 m. A cuboid with the base of 1 m × 1 m is drilled in the centre of each face to the opposite side.

(a) Find the volume of the solid remained.

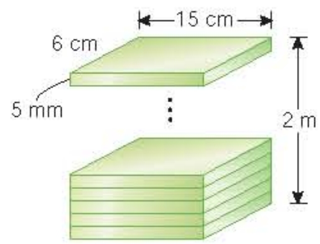
(b) If every 1 m^3 of the solid weighs 1.2 kg, find the weight of the solid remained.



24. Tiles of 15 cm long, 6 cm wide and 5 mm thick each are stacked up to 2 m tall.

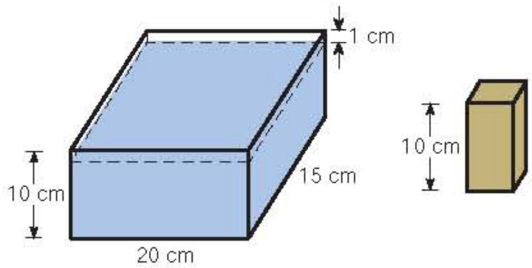
(a) Find the number of tiles.

(b) If all the tiles are used to cover a floor without overlapping each other, find the area covered by the tiles in m^2.

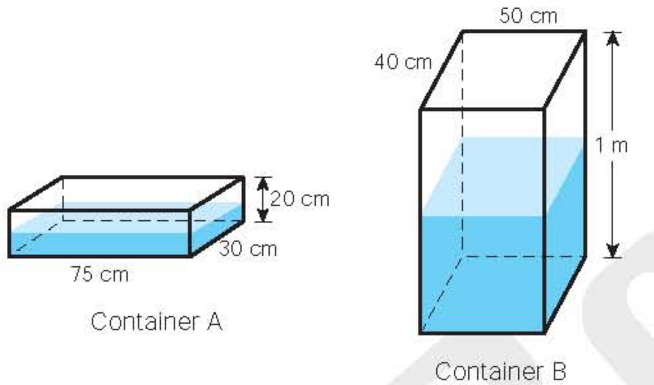


sushi roll 壽司卷 cucumber 青瓜 salmon 三文魚 avocado 牛油果 seaweed 紫菜

25. In the figure, a rectangular tank contains some water and the water level is 1 cm below the top of the tank. After putting a rectangular metal bar with a square base and height of 10 cm in the tank vertically, the water level rises to the top of the tank. What is the base area of the metal bar?



26. The figure shows two rectangular containers which are half filled with water each.



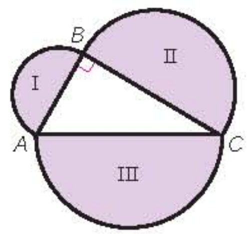
(a) Find the volume of water in Container A.

(b) Find the volume of water in Container B.

(c) If all water in Container A is poured into Container B, find the rise in the water level of Container B.

27. Water flows through a pipe with the diameter of 24 cm at the speed of 2 m/s. How long will it take to fill a cylindrical tank with the diameter of 1.6 m and height of 2 m?

28. In the figure, ABC is a right-angled triangle and $\angle ABC$ is a right angle. Prove that the area of the semi-circle on the hypotenuse is equal to the sum of the areas of the semi-circles on the other two sides.



29. A packet of 250 mL soft drink measures 6.4 cm long, 4.0 cm wide and 10.5 cm thick, correct to 1 decimal place.

(a) **(i)** Find the upper limit and lower limit of the length.

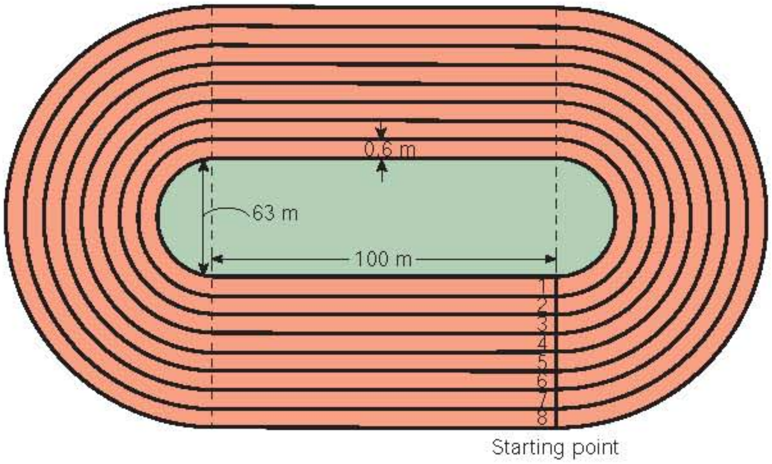
(ii) Find the upper limit and lower limit of the width.

(iii) Find the upper limit and lower limit of the height.

(b) Find the possible range of the volume of the soft drink.

(c) Is the content of 250 mL marked on the package within the range of **(b)**? If not, could you think of a reason?

30. The figure shows the tracks of a sport stadium. The sport stadium is formed by a rectangular field and two semi-circular fields at the two ends with 8 tracks of 0.6 m wide each.

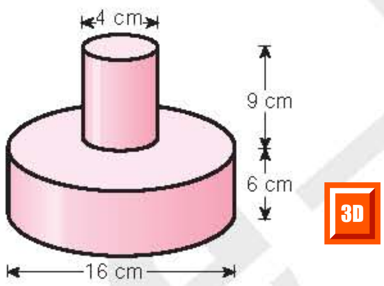


(a) Find the length of the inner edge of track 2.

(b) Find the length of the inner edge of track 8.

(c) Two athletes run at the same speed from the starting point together. One of them runs along the inner edge of track 2 while another runs along the inner edge of track 8. When the athlete on track 2 has just completed a lap, how far is left for the athlete on track 8 to complete a lap?

31. The figure shows a hollow container formed by two cylinders. The cylinder on the top has a base diameter of 4 cm and a height of 9 cm, while the bottom one has a base diameter of 16 cm and a height of 6 cm.

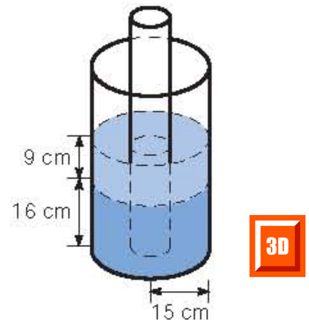


(a) **(i)** Find the capacity of the cylinder on the top.

(ii) Find the capacity of the cylinder at the bottom.

(b) The container is being filled with marbles and water until it is full. If the volume of water in the container is 400 cm^3, find the total volume of the marbles.

32. A cylindrical container with the base radius of 15 cm contains water with depth of 16 cm. After putting a smaller solid cylinder into the container vertically, the water level rises by 9 cm. If the height of the solid cylinder is 35 cm, find its base radius.



MC Question

33. The lateral surfaces of a right prism must be

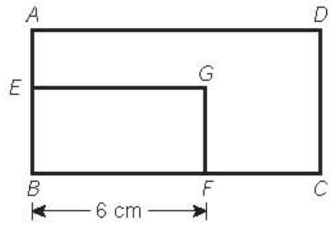
A. squares.

B. rectangles.

C. trapeziums.

D. triangles.

34. In the figure, $ABCD$ and $EBFG$ are two similar rectangles. Their areas are 50 cm^2 and 18 cm^2 respectively. Given that BF = 6 cm, find the length of AE.



A. 2 cm

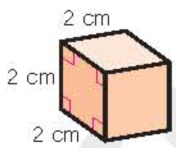
B. 4 cm

C. 5 cm

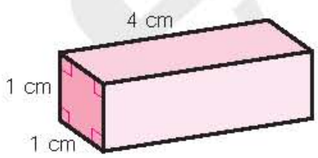
D. 10 cm

35. Which of the following has the largest volume?

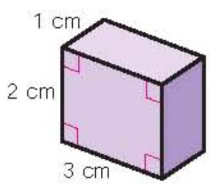
A.



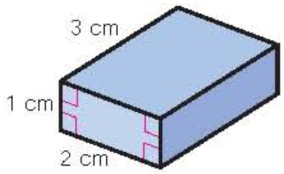
B.



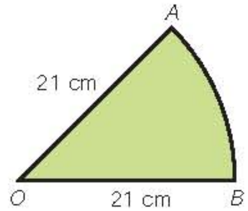
C.



D.



36. Which of the following can be the perimeter of sector OAB?



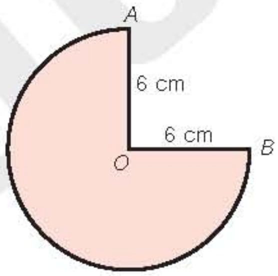
A. 21 cm

B. 42 cm

C. $(42+7\pi)$ cm

D. $(42-7\pi)$ cm

37. The figure shows $\frac{3}{4}$ of a circle. If it is divided into 5 equal parts, what is the area of each part?



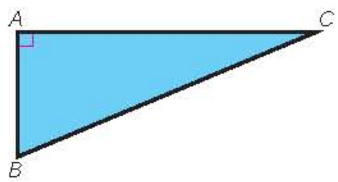
A. $\frac{9}{10}\pi$ cm^2

B. $\frac{27}{5}\pi$ cm^2

C. 9 cm^2

D. 27π cm^2

38. In right-angled triangle ABC, AB = 5 cm and AC = 12 cm, correct to the nearest integer. Find the smallest possible area of ΔABC.



A. 25.875 cm^2

B. 30 cm^2

C. 34.375 cm^2

D. 84.375 cm^2

Problem-solving and Exploring

Hint for the Title Page Question

(a) Find the area of the circle with the radius of 10 cm.

(b) Find the radius of the circle with its area twice that of the red region.

(c) Find the radius of the circle with its area 3 times that of the red region.

(d) Find the radius of the circle with its area 4 times that of the red region.

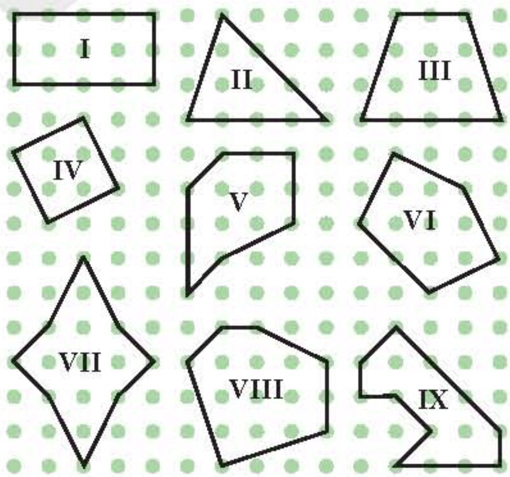
(e) Hence, find the width of each region.

(Give your answers correct to 1 decimal place if necessary.)

Additional Question

1. In the figure, the distance between spots in any two successive rows or columns is 1 unit. Some polygons are drawn with their vertices lying on the spots.

Let i = number of spots inside the polygon,
b = number of spots on the perimeter of the polygon,
A square units = area of the polygon.



(a) Complete the following table.

Polygon	i	b	A
I			
II			
III			
IV			
V			
VI			
VII			
VIII			
IX			

(b) Investigate the relation among i, $\frac{b}{2}$ and A. What have you found?

(c) Draw another diagram to verify your finding in **(b)**.

2. To minimize the chance of causing damage to any products during delivery, efficient utilization of space in packing is necessary.

(a) Work out the size of the box for holding 30 rolls of biscuits as shown in Figure I.

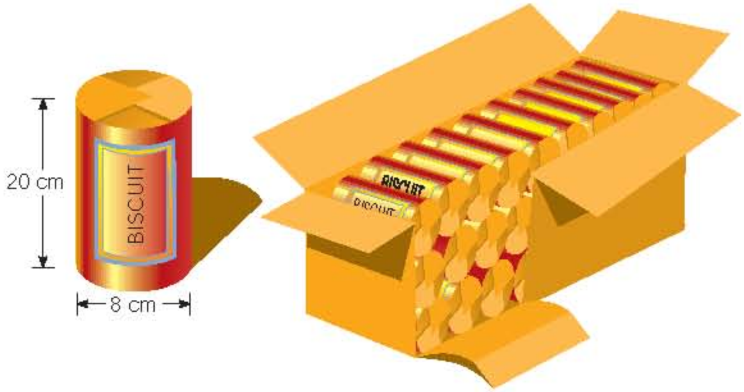


Figure I

(b) A saucepan in Figure II is packed into a box as shown in Figure III.

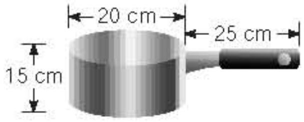


Figure II

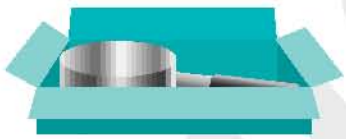


Figure III

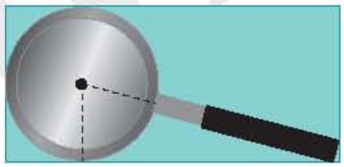


Figure IV

(i) What is the minimum size of the box for 1 saucepan?

[Hint: Put the saucepan into the box diagonally as shown in Figure IV.]

(ii) A carton box in Figure V contains 12 saucepans. What are the dimensions and volume of the box?

Figure V

Origins of Mathematics

The Story of π

1650 BC

The Rhind Mathematical Papyrus (Ancient Egypt) recorded a formula for the area of a circle, from which the value of π was found to be around 3.160 49.

250 BC

Archimedes (Ancient Greece, 287 BC - 212 BC) proved $\frac{223}{71} < \pi < \frac{22}{7}$ using geometric methods.

100 BC

Chou Pei Suan Ching (China) regarded the 'ancient ratio' as 'the circumference is three times the diameter', i.e. $\pi = 3$.

Age of Experiments | **Age of Geometric Methods**

2000 BC

Babylonian mathematicians found that $\pi = 3 + \frac{1}{8}$.

550 BC

The Bible stated that $\pi = 3$.

The 3rd century AD

Liu Hui (China, around the 3rd century AD) used the method '*geyuan shu*' to estimate the value of π. He found $\pi = 3.14$ with a regular 192-gon and later found $\pi = 3.141\ 6$ with a regular 3 072-gon.

The 5th century AD

Zu Chongzhi (China, 429 AD - 500 AD) found that $3.141\ 592\ 6 < \pi < 3.141\ 592\ 7$ using a regular 24 576-gon inscribed in a circle. He gave two approximations of π, $\frac{22}{7}$ (rough ratio) and $\frac{355}{113}$ (detailed ratio).

Age of Experiments (2000 BC - 250 BC)

Long time ago, people usually estimated the value of π by observation or measurement. The Rhind Mathematical Papyrus (around 1650 BC) is the oldest mathematics book in the world, in which the area of a circle is evaluated as the square of $\frac{8}{9}$ of its diameter, i.e. $\pi \approx 3.160\ 49$. On the other hand, the Bible also mentioned descriptions about $\pi = 3$. Since the measuring tools used at that time were very simple, the errors of the values of π obtained were very large.

Age of Geometric Methods (249 BC - 16th Century AD)

The values of π were obtained using geometric methods across this period of time, where the accuracy of the approximate values of π were determined by the choice of computational tools.

Ancient Greece

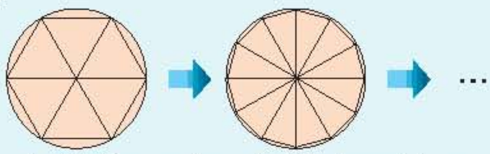
In around 250 BC, Archimedes proved that π was between $\frac{223}{71}$ and $\frac{22}{7}$ using geometric methods.

In around 150 AD, a Greek astronomer Ptolemy found π to be around $\frac{377}{120} \approx 3.141\ 6$, which was more accurate than Archimedes' approximation.

China

In *Fangtian*, the first chapter of *Jiuzhang Suanshu*, the formula for the area of a circle had already been mentioned correctly.

At the end of *Wei Kingdom* and the beginning of *Jin Dynasty*, Liu Hui gave a detailed proof of the formula in Jiuzhang Suanshu. He obtained a more accurate approximation of $\pi \approx \frac{157}{50}$, and the method he used was known as 'geyuan shu'. This method started with the construction of a circle with a radius of 1 unit, followed by a regular hexagon inscribed in the circle. Based on this, the area of a regular $2^n \times 6$-gon (where n is an integer greater than or equal to 0) inscribed in a circle and the approximation of π could be computed.



In the *Northern and Southern Dynasties*, an astronomer Zu Chongzhi made a further breakthrough. He found out that $3.141\ 592\ 6 < \pi < 3.141\ 592\ 7$, which matched the value of π correct to 7 decimal places. This was already a very accurate approximation at that time.

Liu Hui 劉徽　geyuan shu 割圓術　Zu Chongzhi 祖沖之　Fangtian 方田
Jiuzhang Suanshu 九章算術　Wei Kingdom 魏國　Jin Dynasty 晉朝　Northern and Southern Dynasties 南北朝

1593 AD

Francois Viète (France, 1540 AD - 1603 AD) obtained the following formula by making use of an infinite product.

$$\frac{2}{\pi}=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2+\sqrt{2}}}{2}\times\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\times\cdots$$

1706 AD

William Jones (The United Kingdom, 1675 AD - 1749 AD) introduced the Greek letter π to represent the ratio of the circumference of a circle to its diameter.

1949 AD

ENIAC (The USA) was used to compute the value of π, correct to 2 035 decimal places.

Age of Analytic Methods

Age of Computers

Now . . .

1673 AD

Gottfried Wilhelm Leibniz (Germany, 1646 AD - 1716 AD) found the following by using calculus.

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$$

1761 AD

Johann Heinrich Lambert (Germany, 1728 AD - 1777 AD) proved that π was an irrational number.

2002 AD

Yasumasa Kanada (Japan) used a supercomputer to calculate the value of π correct to 1.241 1 trillion decimal places.

India

In around 1400 AD, an Indian mathematician Madhava first made use of an infinite series to obtain a value of $\pi = 3.141\ 592\ 653\ 59$. The accuracy of this value was up to 11 decimal places, which broke the record made by Zu Chongzhi a thousand years ago. Thereafter, the study of mathematical analysis began to develop in the 17th century AD, leading the history of π into a new era.

Age of Analytic Methods (17th Century AD - 1948)

Across this period of time, people began to get rid of the complicated computations of using regular polygons in estimation. They started to use infinite series or infinite products to find the value of π. The following were some examples:

$\frac{2}{\pi}=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2+\sqrt{2}}}{2}\times\frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}\times\cdots$ (found by Francois Viète, a French mathematician, in 1593)

$\frac{\pi}{2}=\frac{2}{1}\times\frac{2}{3}\times\frac{4}{3}\times\frac{4}{5}\times\frac{6}{5}\times\frac{6}{7}\cdots$ (found by John Wallis, a British mathematician, in 1650)

$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ (proposed by Gottfried Wilhelm Leibniz, a German mathematician in 1673)

Using the above formulae, people can obtain a more accurate value of π with less computational steps.

Age of Computers (1949 - Now)

The improvement of computational tools has greatly increased the accuracy in approximate values. In 1949, the first modern computer ENIAC spent only 70 hours to compute the value of π, correct to 2 035 decimal places. In 1960, Daniel Shanks and John Wrench computed the value of π, correct to 100 265 decimal places. In 1967, J. Guilloud computed the value of π, correct to 500 000 decimal places. In 1987, the value of π was found correct to over 29.36 million decimal places. After 2002, the record has exceeded 1.241 1 trillion digits.